NUMERICAL METHODS FOR SINGLE- AND TWO-PHASE FLOWS WITH APPLICATIONS

UERJ - State University of Rio de Janeiro
UFRJ/COPPE - Federal University of Rio de Janeiro

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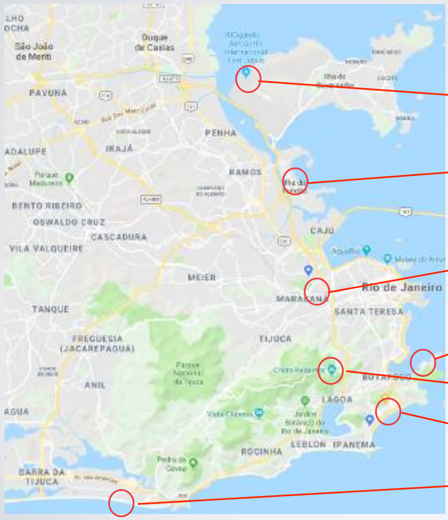
http://www.uerj.br http://2phaseflow.org
http://www.gesar.uerj.br http://gustavorabello.github.io

1st ThermaSMART workshop in Tianjin - China
December 3rd to 5th, 2018

OUTLINE

- Map of Rio de Janeiro;
- Laboratory: GESAR;
- Motivations for 1- and 2-phase flows;
- Numerical Modeling:
  - finite element method - ALE
  - interface modeling;
  - remeshing;
  - demo;
- Results and conclusions.

RIO DE JANEIRO



hotspots:

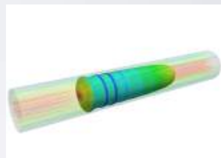
- GIG airport
- UFRJ (federal) — 10min
- UERJ (state) — 30min
- Sugarloaf — 45min
- Corcovado
- Copacabana — 50min
- ICMF

NEXT EVENTS

- International Conference on Multiphase Flows

  19-24 May 2019, Rio de Janeiro, Brazil
  http://www.icmf2019.com.br

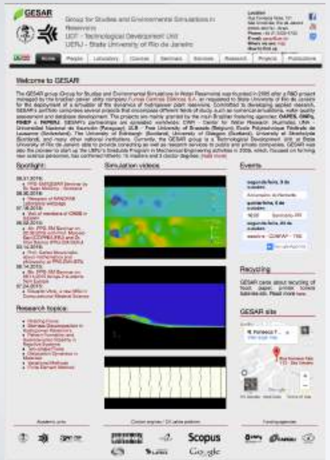
- 4th Workshop on Advances in CFD, LB and MD Modeling of Capillary Two-Phase Flows and Experimental Validation

  16-19 May 2019, Rio de Janeiro, Brazil
  http://2phaseflow.org/publicpages:events

GESAR LAB

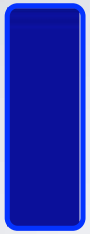
http://www.gesar.uerj.br



Current projects sponsored by Brazilian agencies:

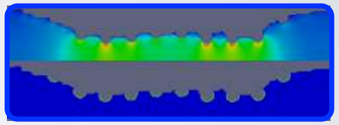
- Flow in Biological systems
- Cooling of electronics
- Two-phase flows
- Phase change
- Rotating disk flows

BACKGROUND

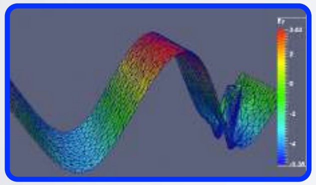


Porous Media
Darcy

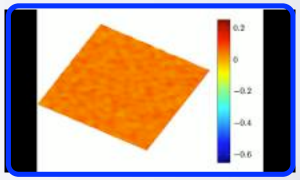
Biologic flows
Navier-Stokes



Electromagnetism
Maxwell



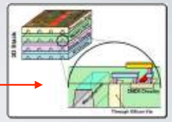
Nano-patterning
Kuramoto-Sivashinsky

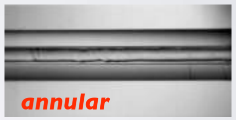


MOTIVATIONS

- cooling of electronics for the new generation of stack microprocessors;

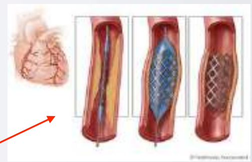
microchannels with different cross-sectional geometries





- aging population and increased level of obesity ensures cardiovascular artery disease (CAD) will remain significant;

stents with different geometries



GOVERNING EQUATIONS

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{c}\cdot\nabla\mathbf{v} = -\frac{1}{\rho(\phi)}\nabla p + \frac{1}{Re}\nabla\cdot\left[\mu(\phi)(\nabla\mathbf{v}+\nabla\mathbf{v}^T)\right] + \frac{1}{Fr^2}\mathbf{g} + \frac{1}{We}\mathbf{f}$$

surface tension

$$\nabla\cdot\mathbf{v} = 0$$

$$\frac{\partial T}{\partial t} + \mathbf{c}\cdot\nabla T = \frac{1}{RePr}\nabla\cdot(k(\phi)\nabla T)$$

ALE formulation: $\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}-\hat{\mathbf{v}})\cdot\nabla\mathbf{v} \begin{cases} \hat{\mathbf{v}} = \mathbf{v} & \rightarrow \text{Lagrangian} \\ \hat{\mathbf{v}} = 0 & \rightarrow \text{Eulerian} \end{cases}$

GOV EQ - AXISYMMETRIC

$$\frac{\partial v_x}{\partial t} + \mathbf{c}\cdot\nabla v_x = -\frac{1}{\rho(\phi)}\frac{\partial p}{\partial x} + \frac{1}{Re}\mu(\phi)\nabla^2 v_x + \frac{1}{Fr^2}g_x + \frac{1}{We}f_x$$

$$\frac{\partial v_r}{\partial t} + \mathbf{c}\cdot\nabla v_x = -\frac{1}{\rho(\phi)}\frac{\partial p}{\partial x} + \frac{1}{Re}\,\mu(\phi)\left(\nabla^2 v_r - \frac{v_r}{r^2}\right) + \frac{1}{We}f_r$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0$$

Laplacian operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

curvature

$$\kappa = \kappa_{2d} + \frac{1}{R} = \kappa_{2d} + \frac{\sin(\theta)}{r}$$

FINITE ELEMENT METHOD

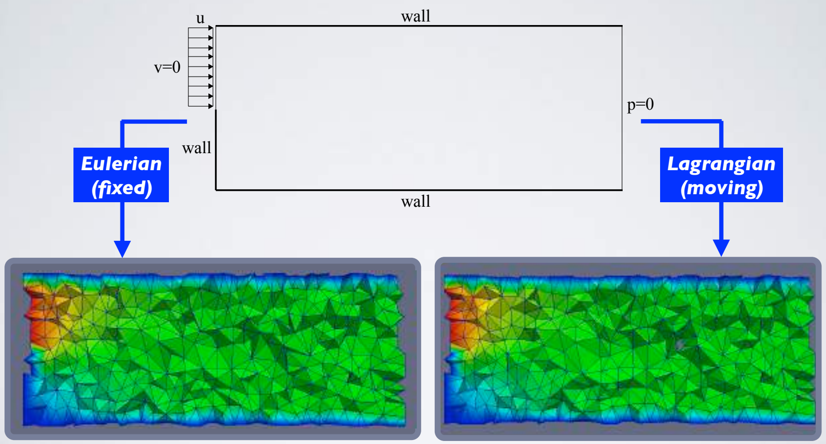
PDE → weak form → ODE → linear system → approximated solution

- PDE: Governing equations
- weak form: Galerkin Method
- ODE: Ordinary Differential Equations
- linear system: Ax=b
- approximated solution: u,v,w,p and T

Code's features:

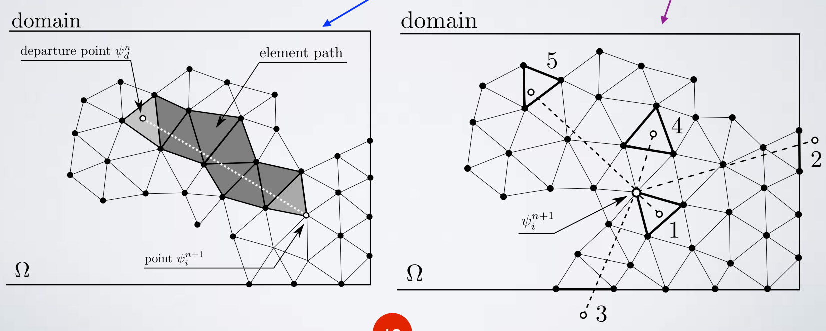
- pressure, diffusive terms → Galerkin method
- convective terms → ALE and SL methods
- time discretization → 1st. order forward difference
- linear systems → Projection method - LU
- surface tension → Geometric

EULERIAN-LAGRANGIAN



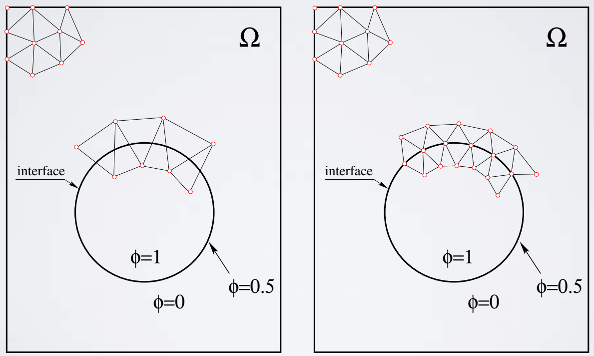
SEMI-LAGRANGIAN

- compute advection using fixed mesh;
- discretization of material derivative: $\frac{D\psi}{Dt} = \frac{\psi^{n+1} - \psi^n}{\Delta t}$
- in ALE context: compute difference between flow field and node motion;
- 2-steps calculation: find departure point (trajectory) and interpolation.



INTERFACE DEFINITION

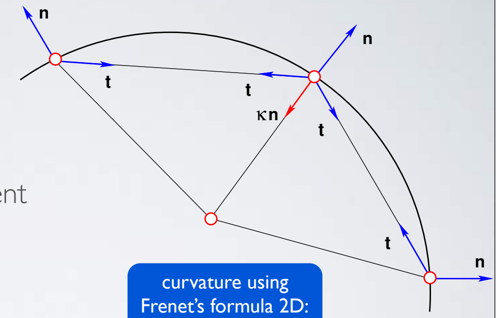
- Eulerian approach: (fixed mesh)
- Lagrangian approach: (moving mesh)



SURFACE TENSION

$$\mathbf{f} = \sigma \kappa \mathbf{n} \delta$$

continuum surface force model
Brackbill and Kothe (1992)



- σ : surface tension coefficient
- κ : curvature
- δ : Dirac delta function
- $\mathbf{n}$: normal vector
- $\mathbf{t}$: tangent vector
- ϕ : marker function (Heaviside)

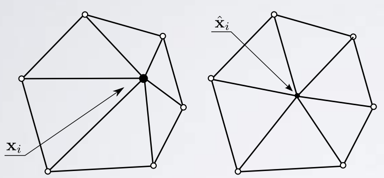
curvature using Frenet's formula 2D:

$$\kappa_i = \frac{\left| \sum_{j=1}^{2} \frac{t_j}{|t_j|} \right|}{\sum_{j=1}^{2} \frac{|d_j - d_i|}{2}}$$

NODES MOTION

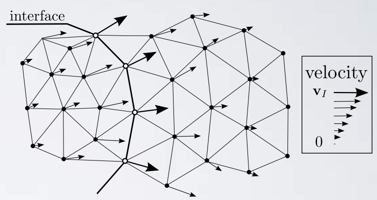
coordinates:

$$\hat{\mathbf{v}}_{e_i} = \frac{\sum_{i \in N_1(j)} e_{ij}^{-1} (\mathbf{x}_j - \mathbf{x}_i)}{dt}$$



velocities:

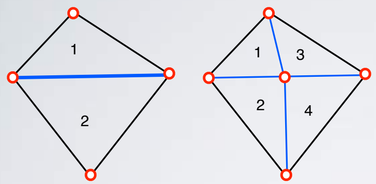
$$\hat{\mathbf{v}}_{v_i} = \frac{1}{n} \sum_{j \in N_1(j)} \mathbf{v}_j$$



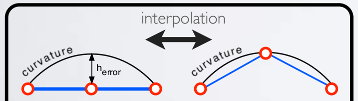
Proposed scheme:

$$\hat{\mathbf{v}}(\mathbf{x}) = \begin{cases} c_1 \mathbf{v} + c_2 \mathbf{v_v} + c_3 \mathbf{v_e} & \text{if } \mathbf{x} \text{ does not belong to the interface} \\ \mathbf{v} - d_1 (\mathbf{v} \cdot \mathbf{t}) \mathbf{t} + d_2 (\mathbf{v_e} \cdot \mathbf{t}) \mathbf{t} & \text{if } \mathbf{x} \text{ belongs to the interface} \end{cases}$$

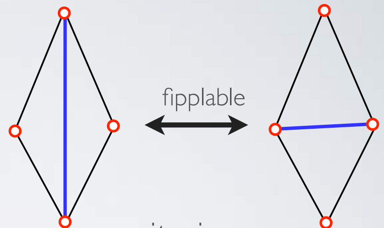
INSERTION



2D view:



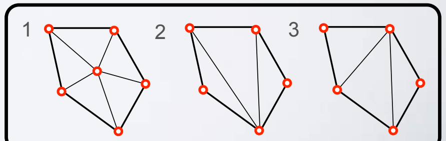
FLIPPING



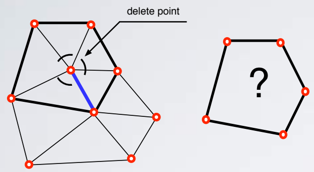
criteria:

aspect ratios, curvature, area and circumcenter

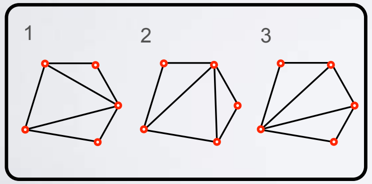
example:



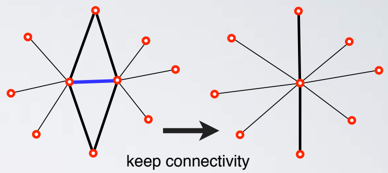
DELETION



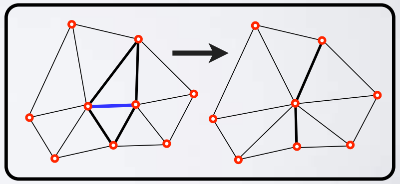
options:



CONTRACTION



example:



DEMO OF ALL METHODS

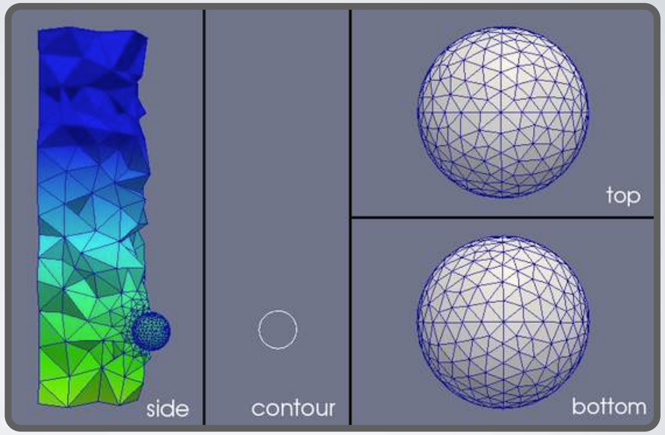
Mesh features:

3d mesh

- insert/flip
- delete
- laplacian smoothing
- Helmholtz Eq.

surface

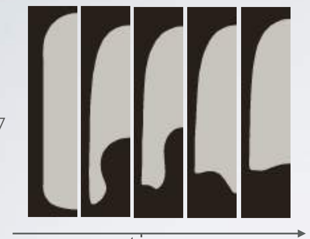
- insert/flip
- delete/contract
- smoothing
- volume conservation

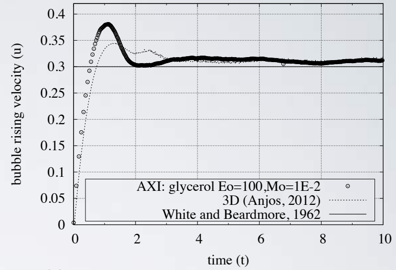


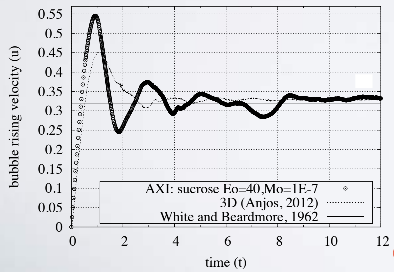
VALIDATIONS

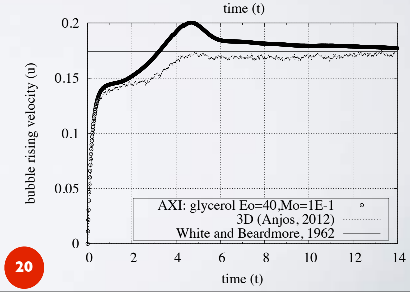
AXI VALIDATIONS

Sucrose solution:
Mo=1e-7
Eo=40

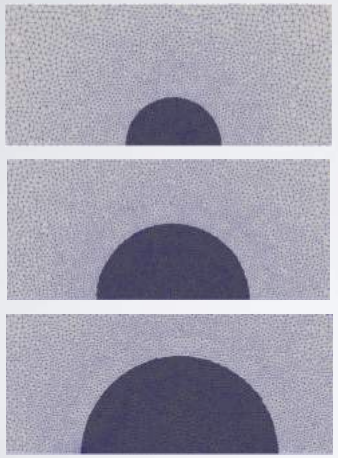






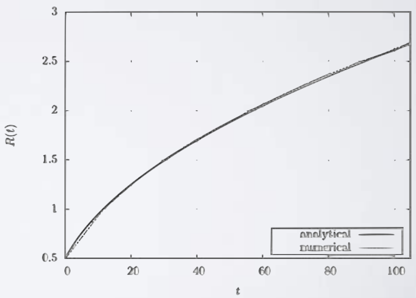


AXI PHASE CHANGE



Evaporating drop
in superheated liquid

Moving mesh method for direct numerical simulation of two-phase flow with phase change (E. Gros, G. Anjos, J. Thome, 2018)



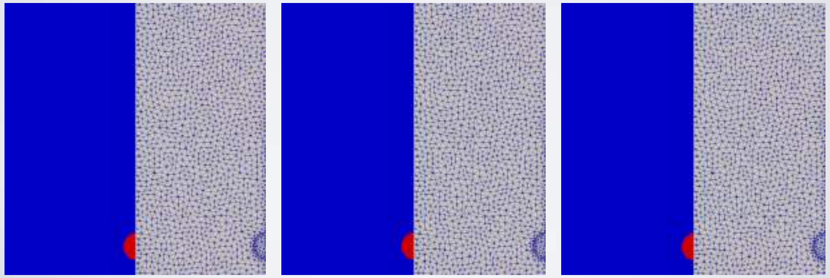
AXI VALIDATIONS

Rising bubbles: interface tracking, surface tension, gravity

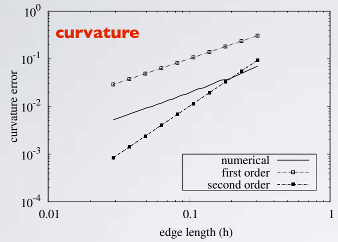
$\rho_{in} = 1.225\text{kg/m}^3 \quad \mu_{in} = 1.78 * 10^{-5}\text{Pa.s}$

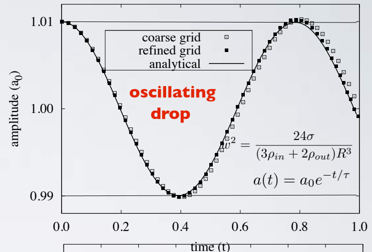
$\rho_{out} = 1350\text{kg/m}^3 \quad We = 115.662$

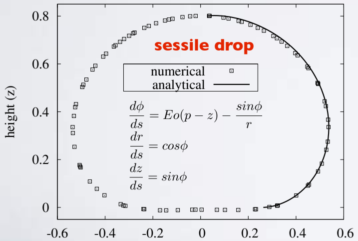
case 1	case 2	case 3
$\mu_{out} = 2.73\text{Pa.s}$	$\mu_{out} = 1.28\text{Pa.s}$	$\mu_{out} = 0.539\text{Pa.s}$
$N = 42.895$	$N = 194.88$	$N = 1091.57$

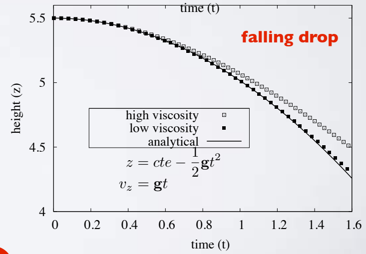


3D VALIDATIONS







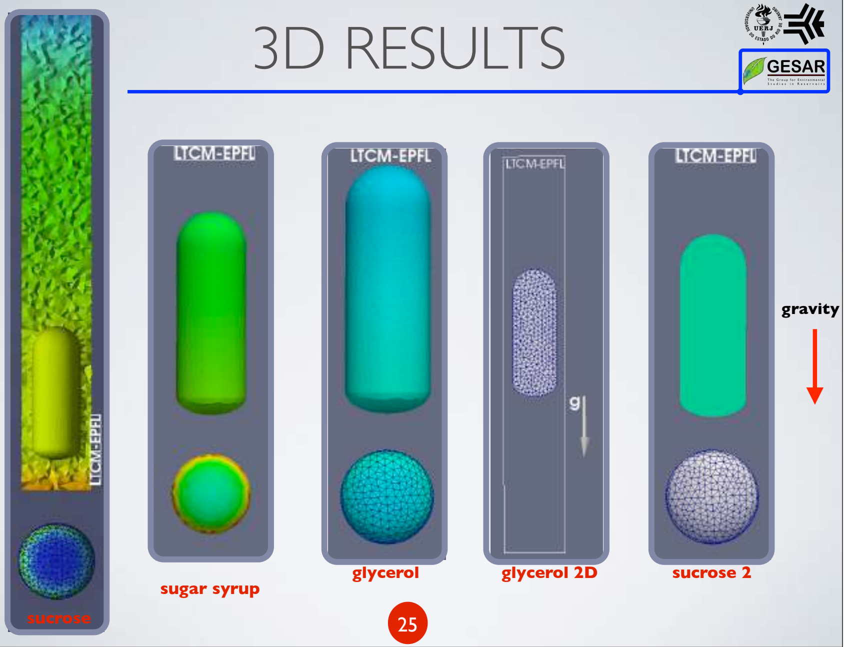


Anjos et al., JCP, 2014 - *published*

RESULTS

3D CASES FOR TAYLOR BUBBLES AND SLUG FLOWS

3D RESULTS

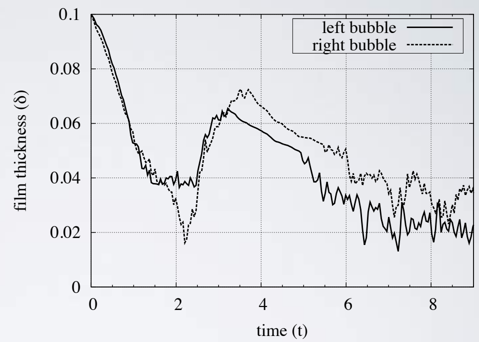


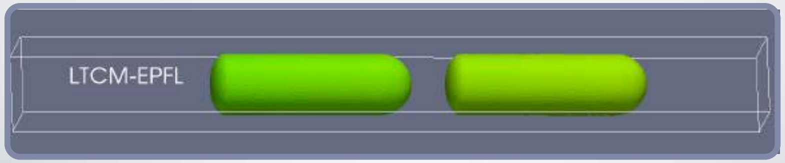
3D RESULTS

parameters:

fluid: R1234ze
D=0.0001m
sigma=8.5156 mN/m
mu_in = 12.464E-06 Ns/m^2
mu_out = 202.71E-06 Ns/m^2
rho_in = 25.89 kg/m^3
rho_out = 1157.4 kg/m^3

domain: DxDx10D
mesh elements: 140000 tetrahedrons
mesh nodes: 22000 nodes
surface elements: 22000 triangles
surface nodes: 11000 nodes





RESULTS

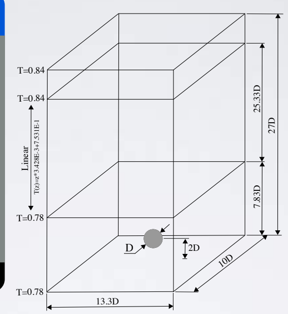
BUBBLE RISING IN HEATED LIQUID

3D RESULTS

parameters:

fluid: air - water-sugar (low viscosity)
d = 5.97mm
mu_in / mu_out = 0.179
rho_in / rho_out = 0.0012
We = 4.90
Fr = 1
Re = 1455

Domain: 13.3Dx10Dx27D
mesh elements: 40000 tetrahedrons
mesh nodes: 6100 nodes
surface elements: 9500 triangles
surface nodes: 4000 nodes



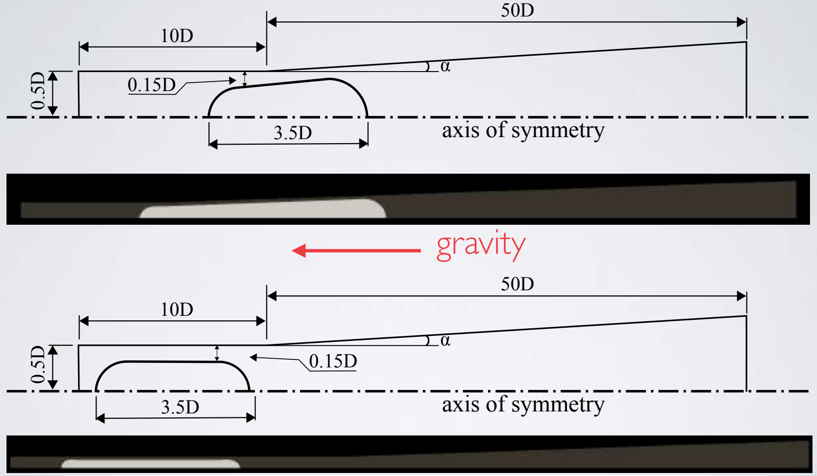
$$T_{ini} = \begin{cases} 0.78 & y \leq 7.83D \\ z * 3.428E-3 + 7.531E-1 & 7.83D < y < 25.33D \\ 0.84 & y \geq 25.33D \end{cases}$$

Mixing of thermally stratified water layer by a free rising wobbling air bubble, Chemical Engineering Science (Zun et al, 2012)

RESULTS

TAYLOR TO SPHERICAL CAP TRANSITION IN DIVERGENT CHANNEL

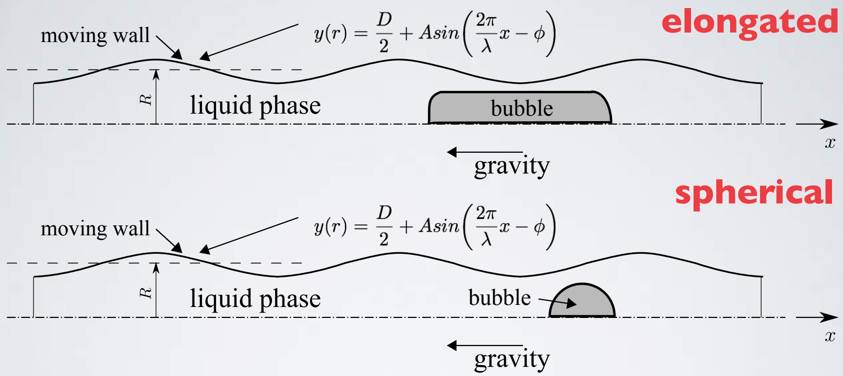
AXI RESULTS: DW+G



RESULTS

SINUSOIDAL CHANNEL

GEOMETRIES

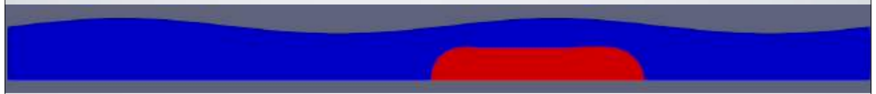


wall moves as: $y(r) = \frac{D}{2} + A sin\left\{\frac{2\pi}{\lambda}(x - x_c) - \phi\right\}$

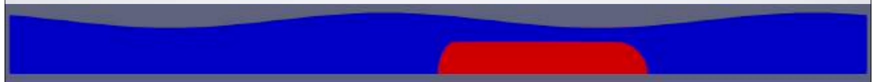
where x_c is bubble's centroid position

RESULTS: BUBBLE SHAPE

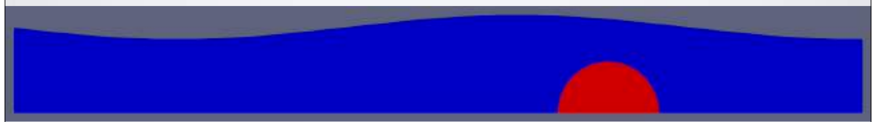
DEGG3 - Diethylene–glycol, higher inner viscosity



DEGG11 - Diethylene–glycol



DEGG3 - Diethylene–glycol, higher inner viscosity

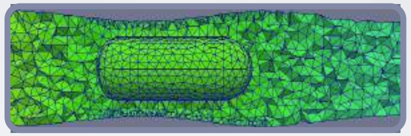


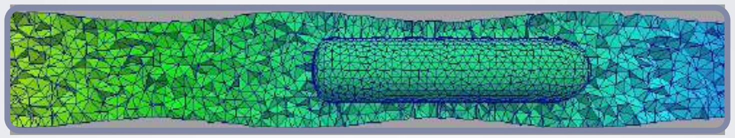
PROJECT OUTPUT

- Strong numerical background for several applications, including cooling of electronics, rotating disk, pattern formation, corrosion, electromagnetism etc.
- Development of analytical mathematical models and inverse problems to provide reliable estimates of physical parameters in single- and two-phase flow systems;
- Establish a sustainable cooperation between researchers from PPG-EM/UERJ, COPPE/UFRJ (Brazil) and ThermaSMART partners for single- and two-phase flows problems, including porous media.

THANK YOU!

http://gustavorabello.github.io

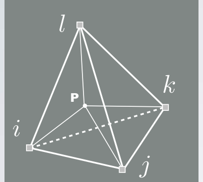




THANKS TO: THERMASMART Thermal Management of Microprocessors; FAPERJ Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

FINITE ELEMENT METHOD

volume coordinates:



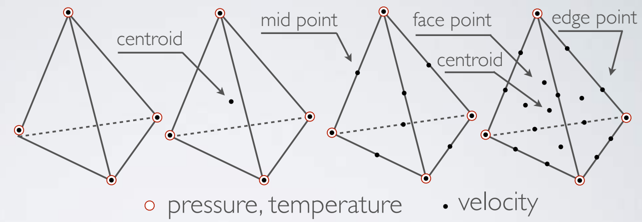
$$L_i = \frac{V_i}{V} \quad L_j = \frac{V_j}{V}$$

$$L_k = \frac{V_k}{V} \quad L_l = \frac{V_l}{V}$$

$$x = L_i x_i + L_j x_j + L_k x_k + L_l x_l$$

$$y = L_i y_i + L_j y_j + L_k y_k + L_l y_l$$

$$z = L_i z_i + L_j z_j + L_k z_k + L_l z_l$$



finite elements:

element:	nodes:	LBB:	lin sys:
linear	4	unstable	small
mini	5	stable	moderate
quad	10	stable	large
quad+bubble	11	stable	large
cubic	20	stable	very large

Anjos et al., JES, 2012