Development of a two-phase compressible Euler numerical solver with applications to shock wave interactions with gas bubbles

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Two Phase Compressible Euler Solver

In each phase solve the compressible Euler equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0$$

$$\frac{\partial}{\partial t}(\rho \underline{u}) + \nabla \cdot (\rho \underline{u}\,\underline{u} + p\underline{I}) = 0$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (\underline{u}(E + p)) = 0$$

$$E = \rho e + \frac{1}{2}\rho|\underline{u}|^2$$

$$p = B\left(\frac{\rho}{\rho_0}\right)^{\gamma} - B + A$$

$$p = (\gamma - 1)\rho e$$

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Conservative Form

$$\frac{\partial \underline{U}}{\partial t}+\frac{\partial \underline{F}(\underline{U})}{\partial x}+\frac{\partial \underline{G}(\underline{U})}{\partial y}+\frac{\partial \underline{H}(\underline{U})}{\partial z}=0; \qquad \underline{U}=(\rho, \rho u, \rho v, \rho w, E)^T$$

$$\underline{F}(\underline{U})=\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ \rho uw\\ (E+p)u\end{pmatrix}, \qquad \underline{G}(\underline{U})=\begin{pmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ \rho vw\\ (E+p)v\end{pmatrix}, \qquad \underline{H}(\underline{U})=\begin{pmatrix}\rho w\\ \rho uw\\ \rho vw\\ \rho w^2+p\\ (E+p)w\end{pmatrix}$$

Compressible Euler Solver in each Phase

- Method of lines applied to separate spatial and temporal integration
- Spatial fluxes - 3rd order ENO-ROE method employed with similarities to the Marquina flux splitting technique
- In time a 3rd order TVD Runge-Kutta method employed
- Timesteps calculated adaptively to ensure CFL condition meet

Interface Evolution

Interface captured using a level set function $\varphi(\underline{r}, t)$

$$\frac{\partial \varphi}{\partial t} + v \cdot \nabla \varphi = 0$$

Reinitialise to a sign distance function

$$\frac{\partial \varphi}{\partial \tau} + \hat{S}(\varphi_0)\left[\left|\nabla \varphi\right| - 1\right] = 0$$

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Original Ghost Fluid Method

- Gas/Gas Systems
- Pressure and velocity from real fluid of phase 1 copied into the ghost fluid of phase 2 and vice versa
- Density extrapolated from real fluid into ghost fluid for a given phase

$$\frac{\partial I}{\partial \tau} + \underline{n} \cdot \nabla I = 0$$

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Modified Ghost Fluid Method

- Air pressures and water normal velocities dictate interface values
- Gives greater dissipation at the interface, thus offering greater stability in order to counter numerical issues associated with the stiff nature of the Tait equation

Ghost Air Cells

- Copy real values of the normal velocity components of water into each node in the ghost region;

- Extrapolate pressure & tangential components of velocity from the first set of cells adjacent to the interface in the real air region across the interface;

- Set density values using the equation $\rho = \hat{\rho}\left(\frac{p_0}{\hat{p}}\right)^{1/\gamma}$ where $\hat{\rho},\ \hat{p}$ obtained by extrapolating the density and pressure values from the second set of cells relative to the interface in the real air region

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Ghost Water Cells

- Copy real air pressure and tangential velocity values into each node in the ghost water region;

- Extrapolate normal components of velocity from the first set of cells adjacent to the interface in the real water region;

- Set density values using the equation $\rho = \hat{\rho}\left(\frac{p_0 + B_2}{\hat{p} + B_2}\right)^{1/\gamma}$

where $\hat{\rho}$, $\hat{p}$ obtained by extrapolating the density and pressure values from the second set of cells relative to the interface in the real water region

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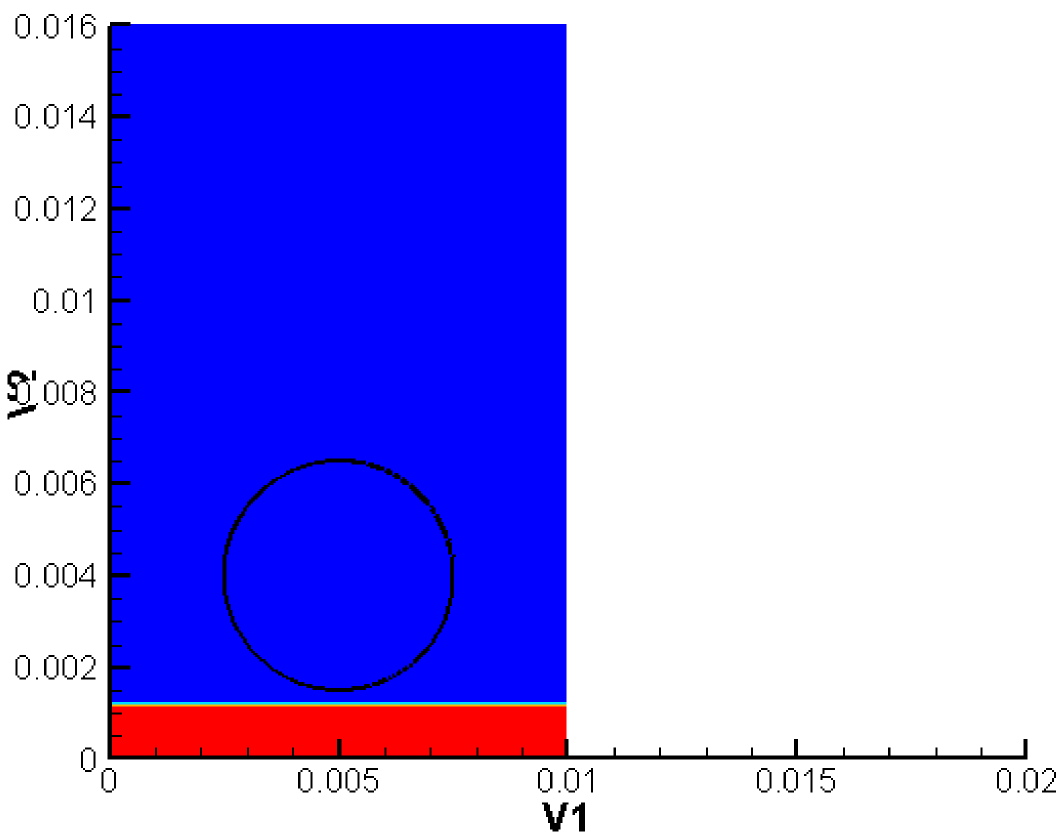
Impact of a helium bubble in air by a shock wave



Pressure contours

2D Cartesian geometry

Initial radius 5mm

Shock strength 0.15698MPa

401x641 grid

3rd order ENO-ROE

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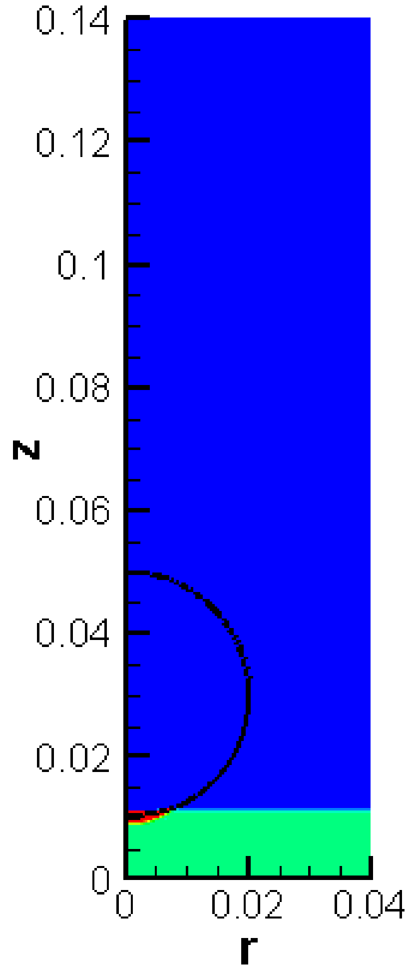
Impact of a krypton bubble in air by a shock wave



Pressure contours

Axisymmetry geometry

Initial radius 20mm

Shock strength 0.15698MPa

401x1601 grid

1st order ENO-ROE

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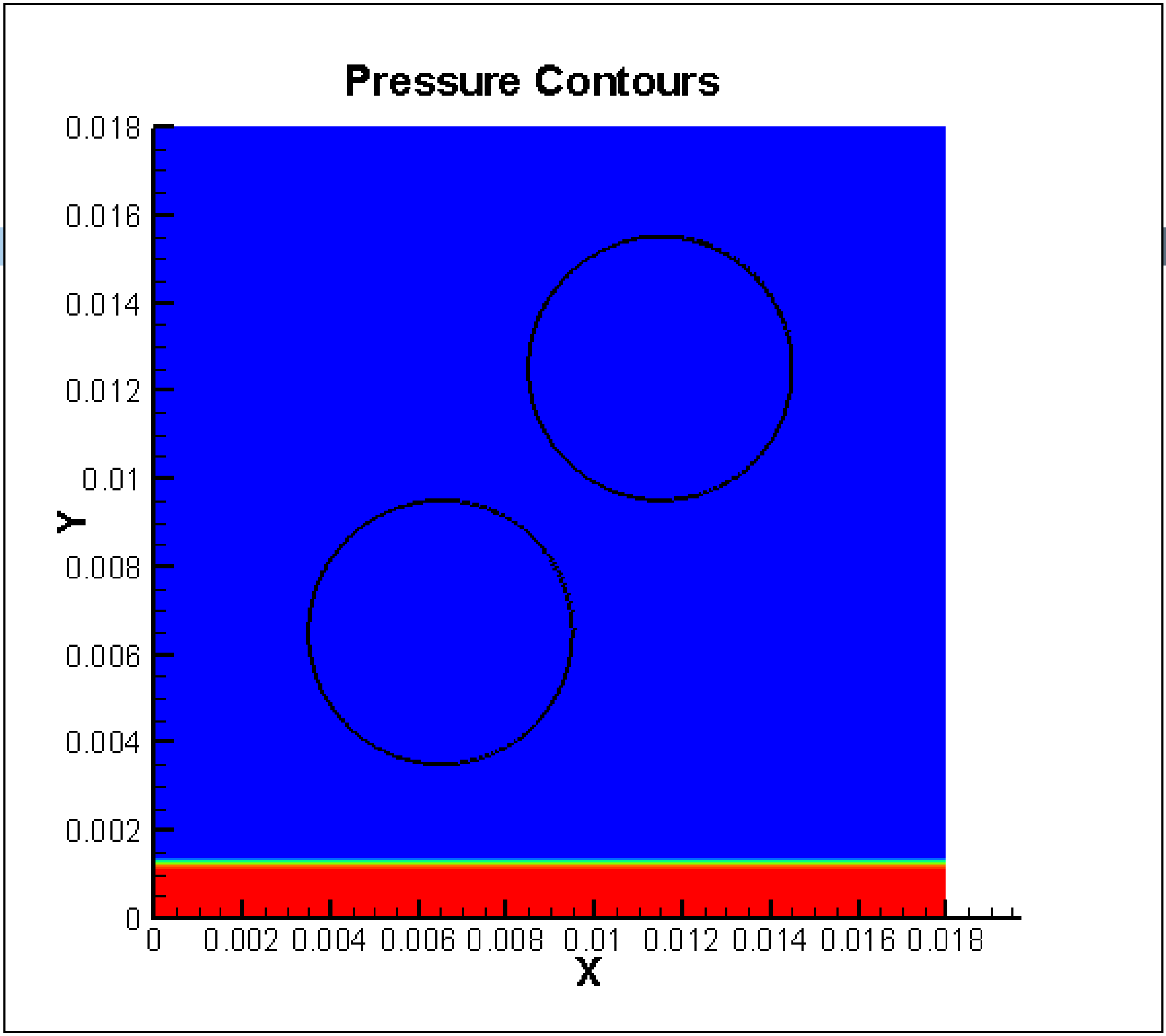

2D Cartesian geometry

Shock strength 100MPa

2 air bubbles in water

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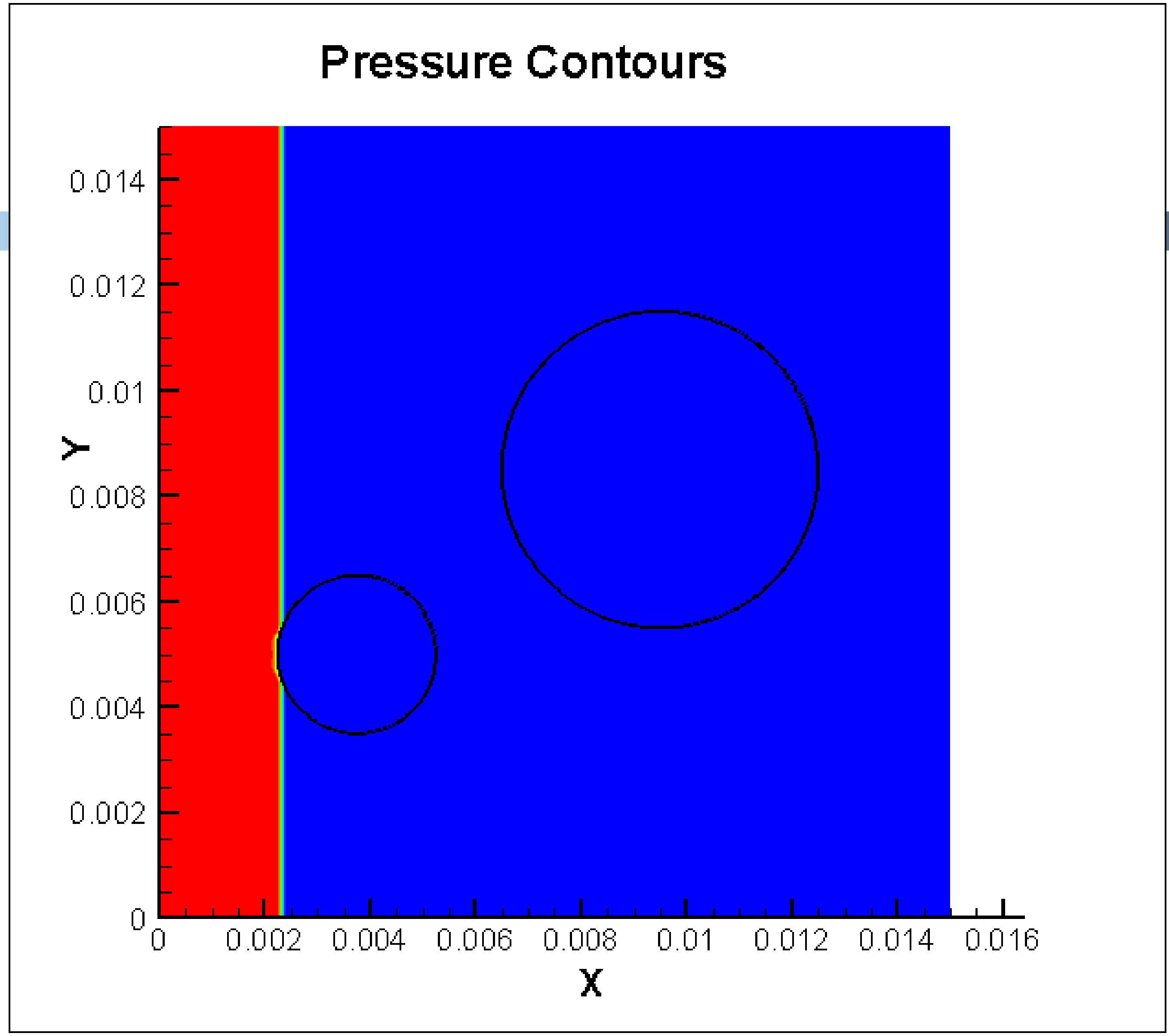

2D Cartesian geometry

Shock strength 200MPa

2 air bubbles in water

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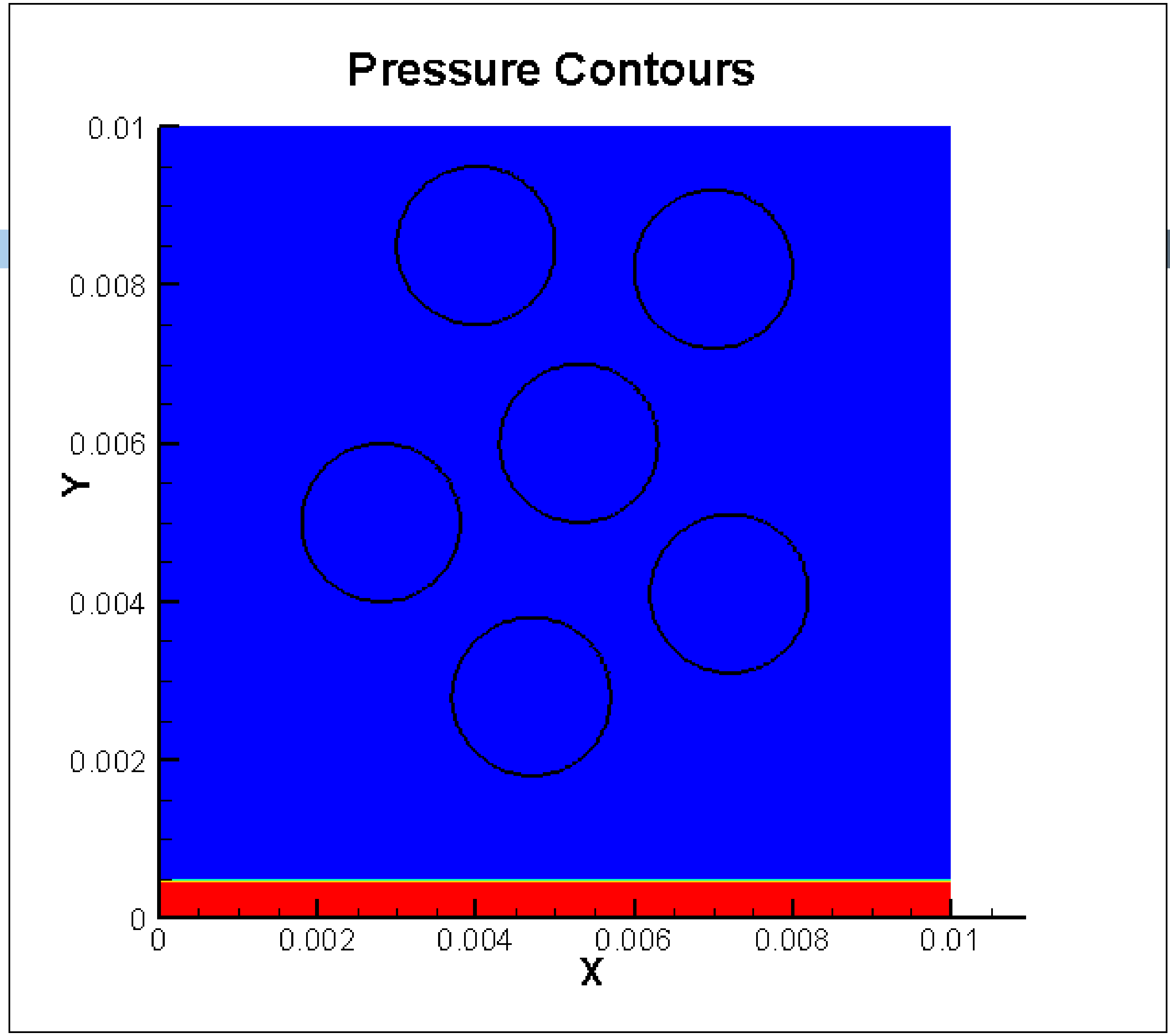
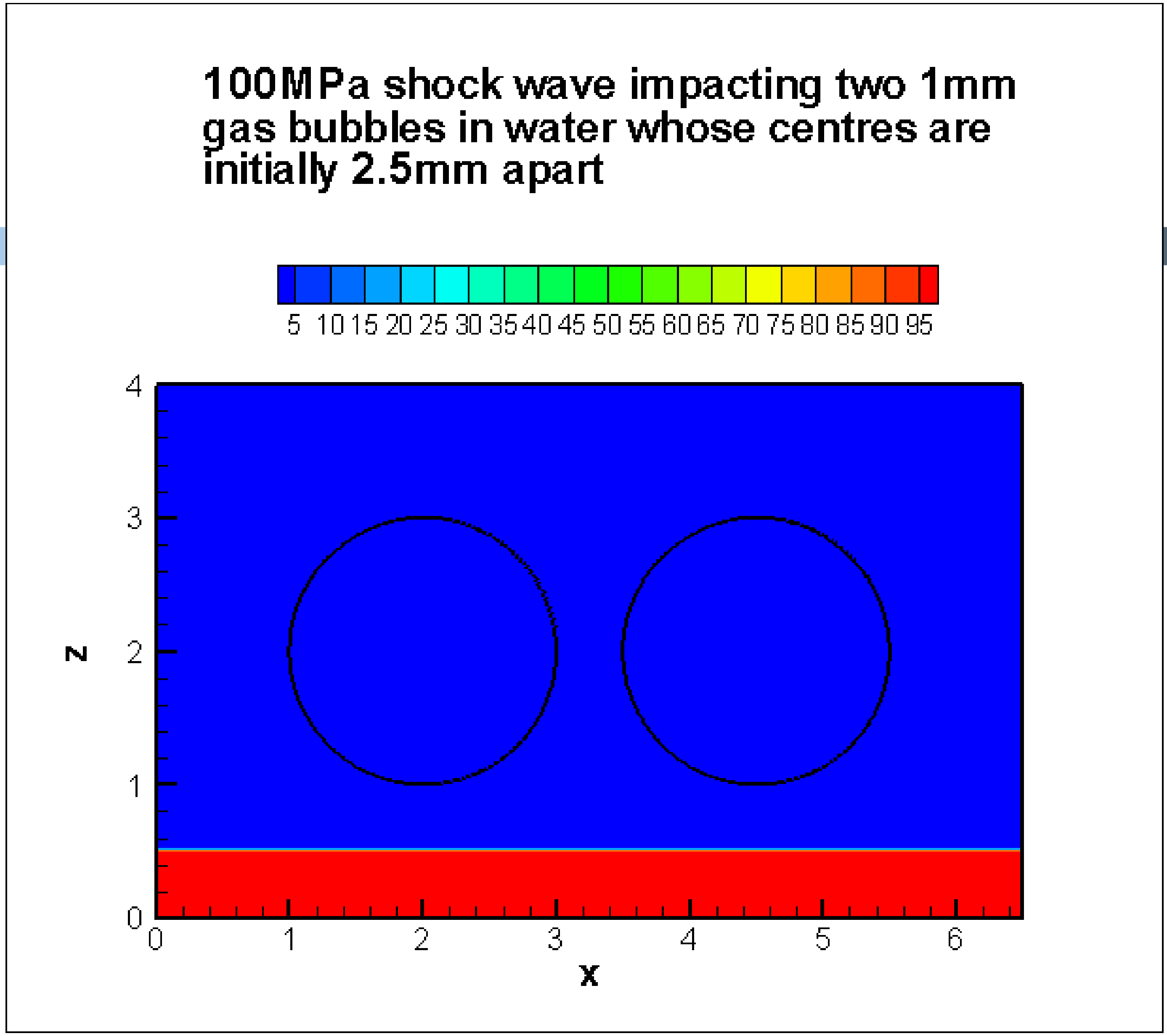


2D Cartesian geometry

Shock strength 10MPa

6 air bubbles in water



3D Cartesian geometry

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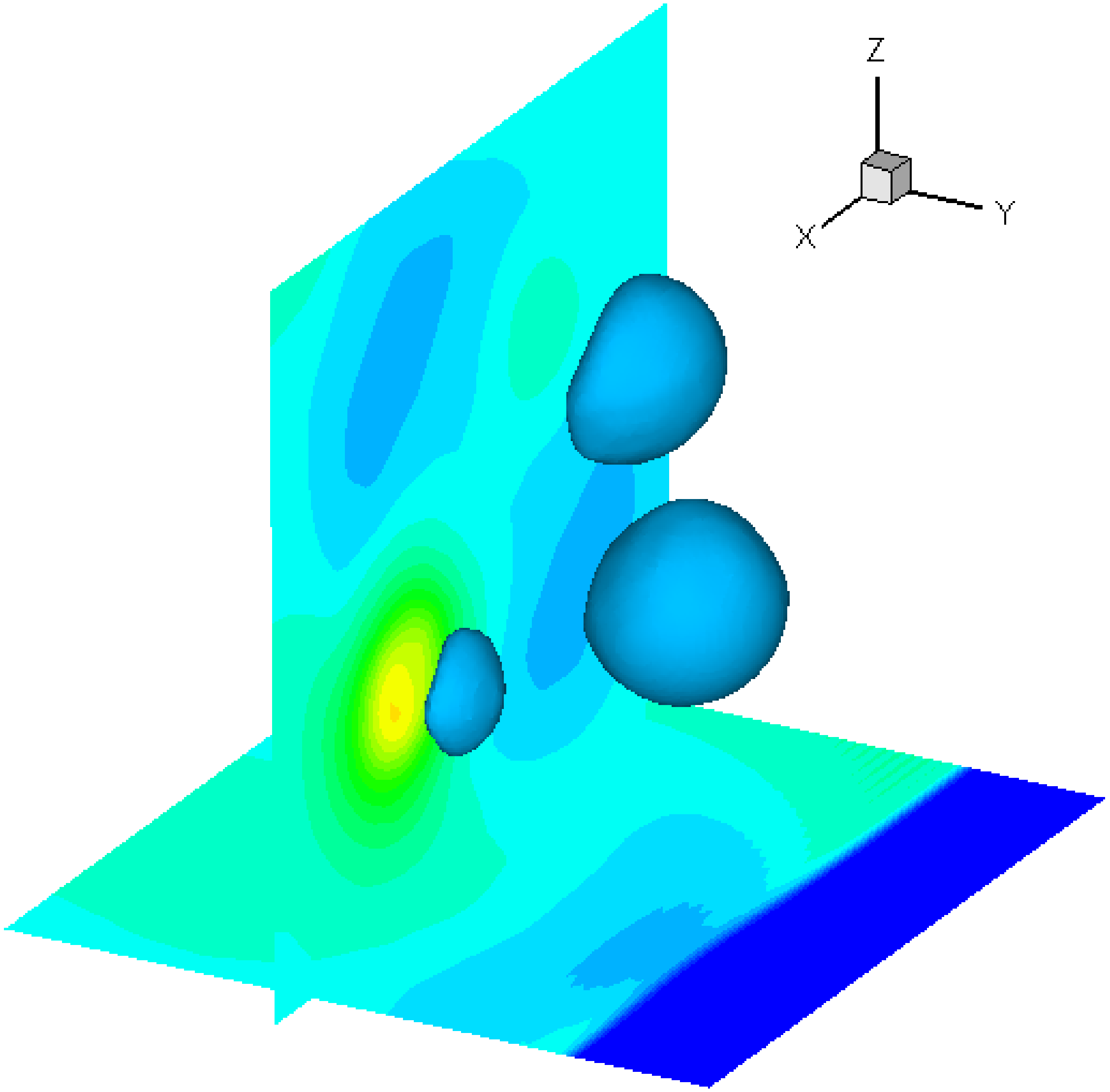

Three
air
Bubbles

400MPa
shock
wave

3D Cartesian geometry

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Issues

- accuracy of interface conditions
- conservation of mass
- numerical instability for strong shocks
- effective parallelisation
- extension of applied interface conditions to include phase change or chemical reactions

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Modified Ghost Fluid Method

- A Riemann problem is defined and solved approximately to predict the interface state
- The predicted interface state is used to define ghost fluid states by interpolation.

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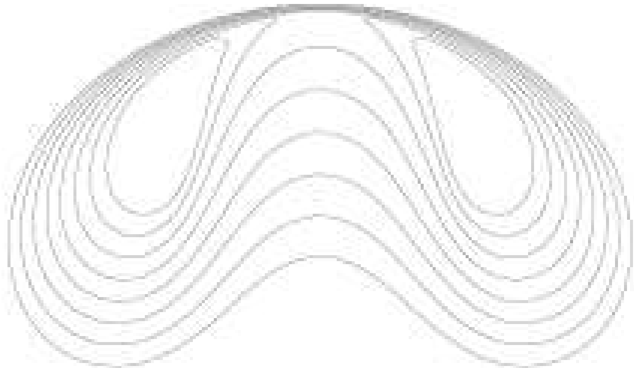

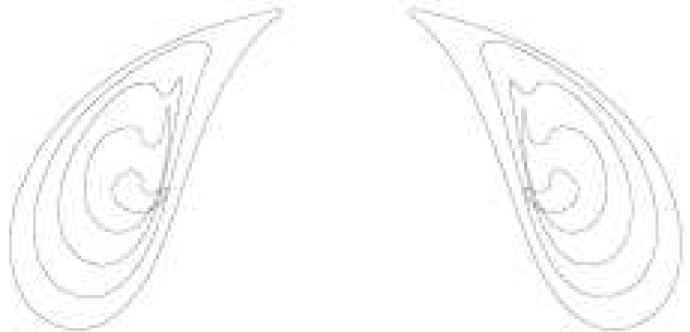

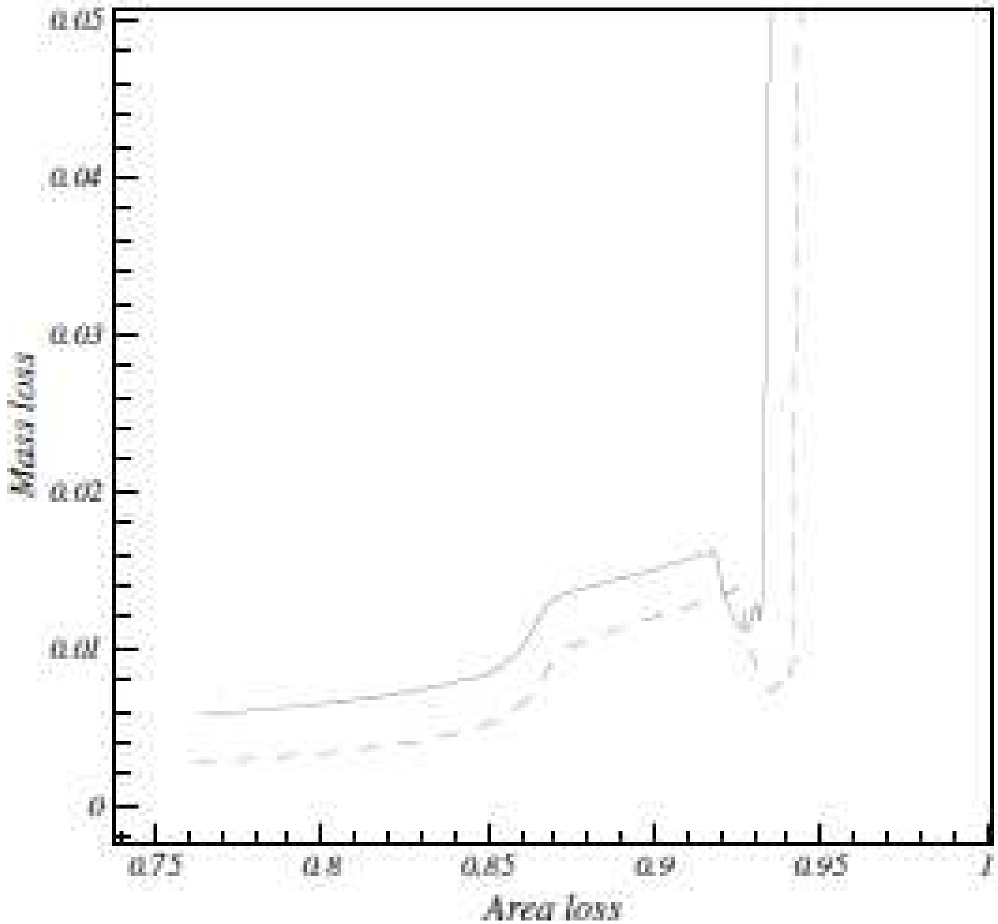

2D Geometry

Initial radius
3mm

400MPa shock

1201x1201 grid

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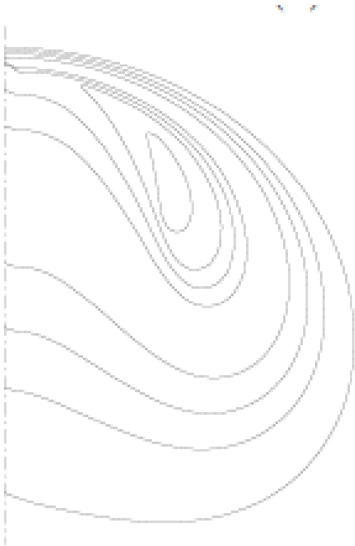

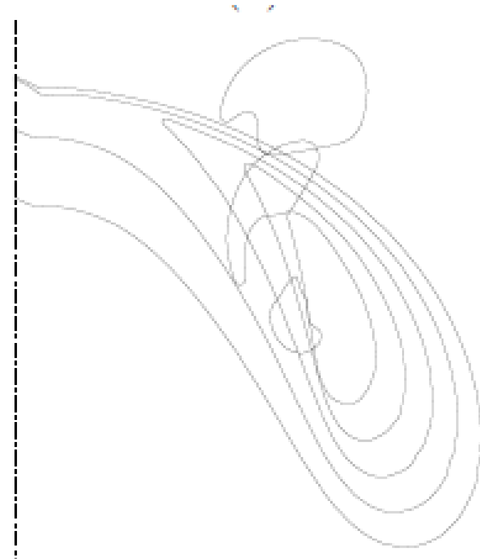

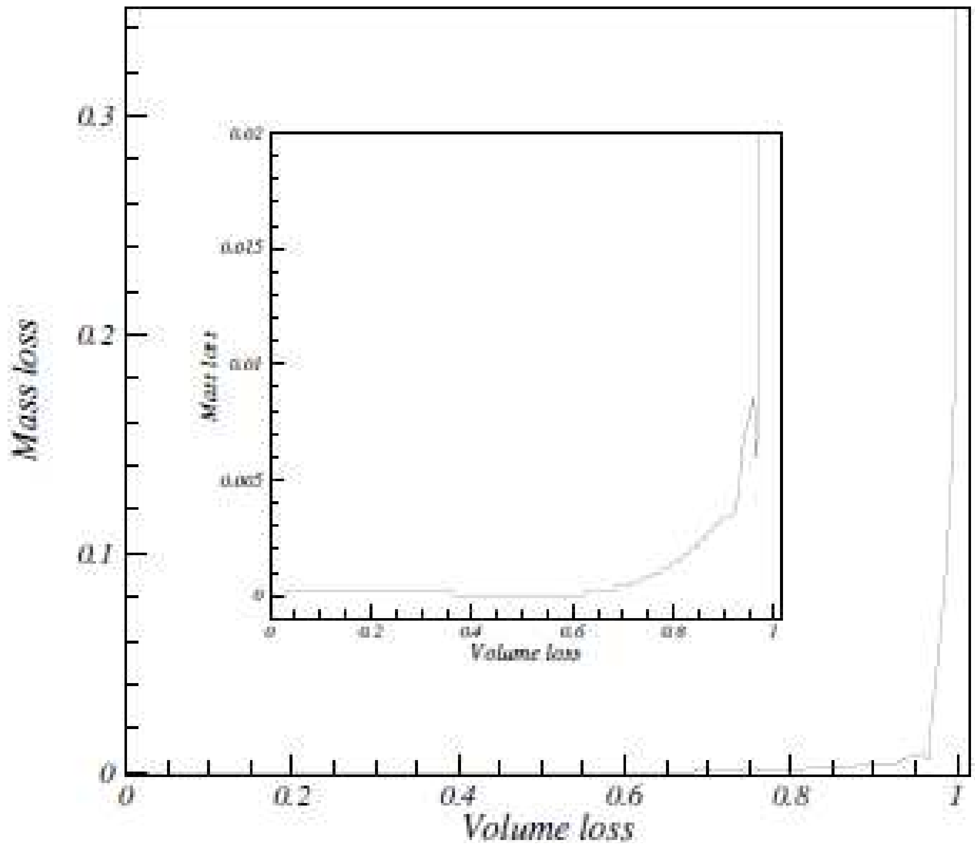

Axisymmetry

Initial radius 1mm

200MPa shock

801x1601 grid

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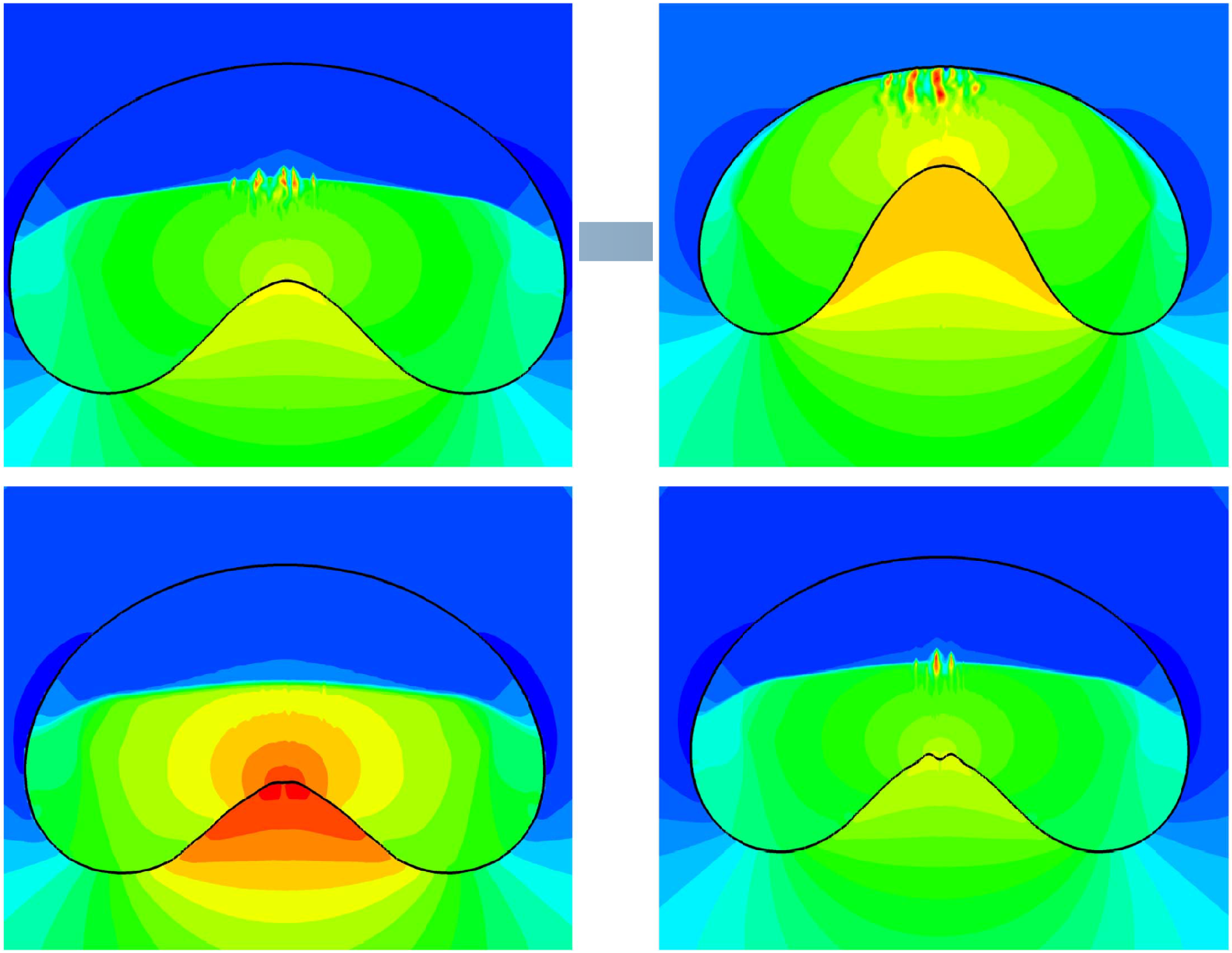

2D Cartesian geometry

3mm air in water

Shock strength 0.4GPa

y-velocity contours

(a) 1201x1201 grid, *t*=6.0μs;
(b) 1201x1201 grid, *t*=6.6μs;
(c) 401x401 grid, *t*=6.0μs;
(d) 801x801 grid, *t*=6.0μs.

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Conclusions

Much still to be done!!!

References

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