CHAOTIC ORBITS OF TUMBLING ELLIPSOIDS IN VISCOUS AND INVISCID FLUIDS

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MOTIVATION

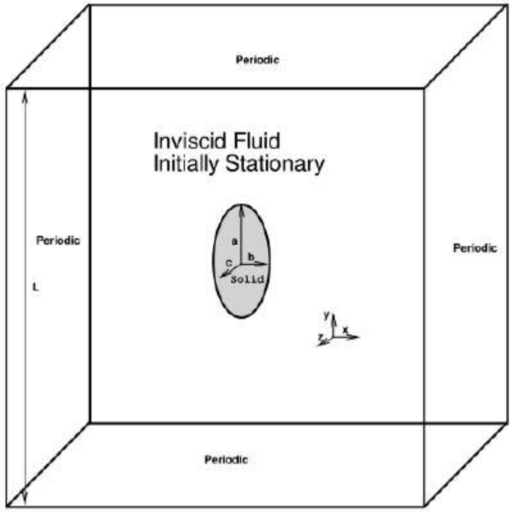
Kozlov & Oniscenko (1982) showed that general ellipsoids can show chaotic motion in inviscid flow

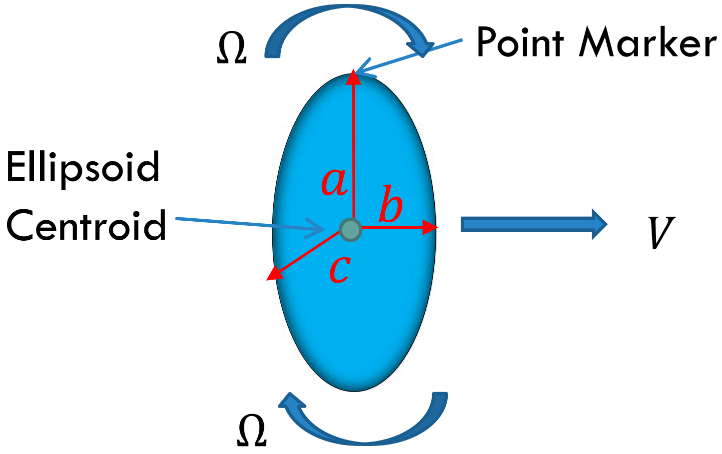
Objectives:

- Can chaotic behaviour be seen in viscous flows using DNS
- Can chaotic dynamics be used to enhance mixing.

MOTION OF ELLIPSOID IN A FLOW

Problem Set-up





Chaotic motion ***only*** under conditions of non-integrability of Kirchhoff's equations (Kozlov & Oniscenko, *Sov. Math. Dokl* 1982)

General Ellipsoid: $a \neq b \neq c$

Domain Scale: $\mathrm{L} = 512a$

$$Re_p = \frac{2Va}{\nu}$$

$$\mathrm{E} = \frac{k_t}{k_r} = \frac{mV^2}{I\Omega^2}$$

MOTION OF ELLIPSOID IN INVISCID FLOW

$$\begin{pmatrix} \boldsymbol{L} \\ \boldsymbol{P} \end{pmatrix} = \begin{pmatrix} I & D \\ D^T & M \end{pmatrix} \triangleq \mathcal{M} \begin{pmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{pmatrix}$$

$$\dot{\boldsymbol{L}} = \boldsymbol{L} \times \boldsymbol{\omega} + \boldsymbol{P} \times \mathbf{v} + \boldsymbol{T_s}$$
$$\dot{\boldsymbol{P}} = \boldsymbol{P} \times \boldsymbol{\omega} + \boldsymbol{F_s}$$

$$\boldsymbol{q} = (\boldsymbol{L}, \boldsymbol{P})$$
$$\mathcal{H}(\boldsymbol{q}) = \frac{1}{2} \boldsymbol{q} \cdot \mathcal{M}^{-1} \boldsymbol{q}$$

$\boldsymbol{L}$ – Angular Momentum

$\boldsymbol{P}$ – Linear Momentum

$\boldsymbol{\omega}$ – Angular Velocity

$\mathbf{v}$ – Linear Velocity

THE LIMITS OF INTEGRABILITY

The Kirchhoff equations can be view as a Lie algebra, SE(3)

Casimir functions are:

$$C_1 = \boldsymbol{P} \cdot \mathrm{L}; \qquad C_2 = \|P\|^2$$

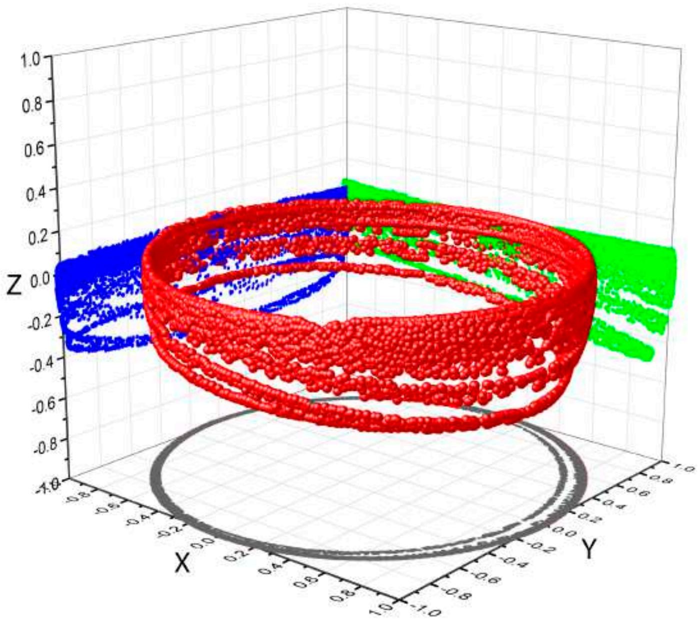
In the general case $C_2 \neq 0$

$\mathcal{H}$ restricts system to a level set (coadjoint orbit)

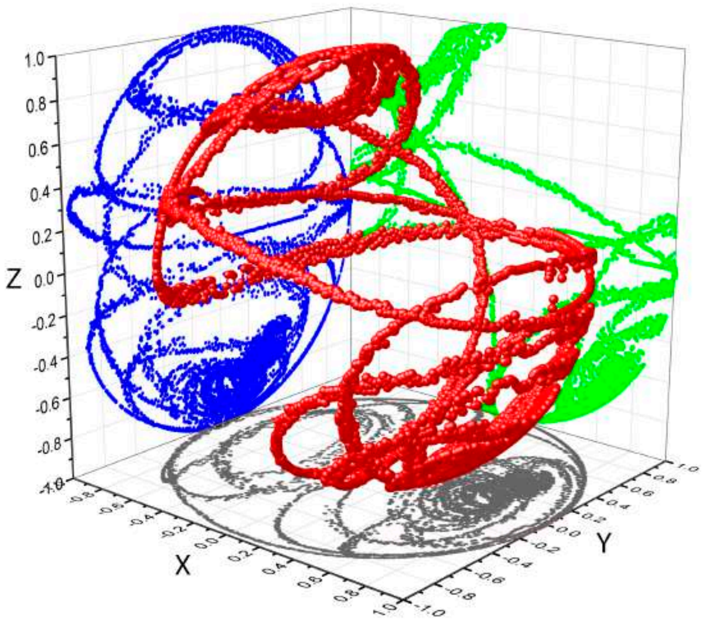
For the equations to be integrable additional conversed quantity is required

ELLIPSOID ORBITS IN INVISCID FLUID

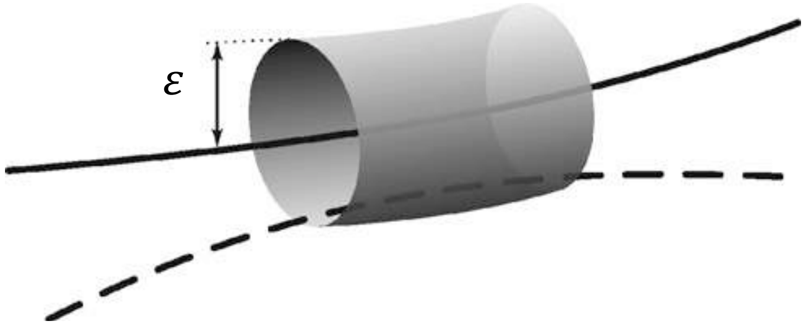
Periodic



Chaos



RECURRENCE QUANTIFICATION ANALYSIS (RQA) OF ELLIPSOID MOTION



$$R(i,j) = \begin{cases} 1 \; if \; L_\infty(\vec{x_i}, \vec{x_j}) \le \varepsilon \\ 0 \; otherwise \end{cases}$$

Recurrence Rate (RR)

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R(i,j)$$

DET Determinism of the system.

$$DET = \frac{\sum_{\ell=\ell_{min}}^{N} \ell P(\ell)}{\sum_{\ell=1}^{N} \ell P(\ell)}$$

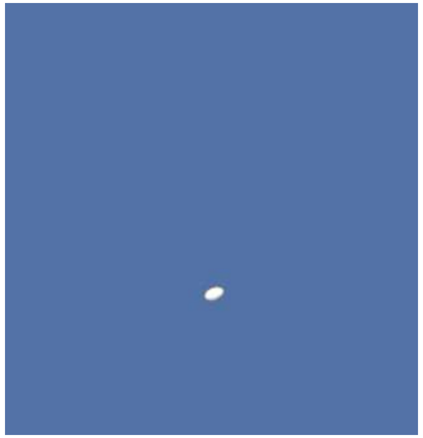
The Lyapunov exponent and Rényi Entropy of the system encoded in recurrence plot.

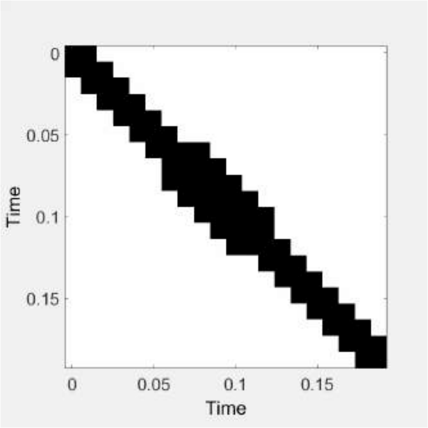
$$ENTR = -\sum_{\ell=\ell_{min}}^{N} p(\ell) \ln p(\ell)$$

ELLIPSOID ORBITS AT DIFFERENT ENERGY RATIOS

Periodic Orbit

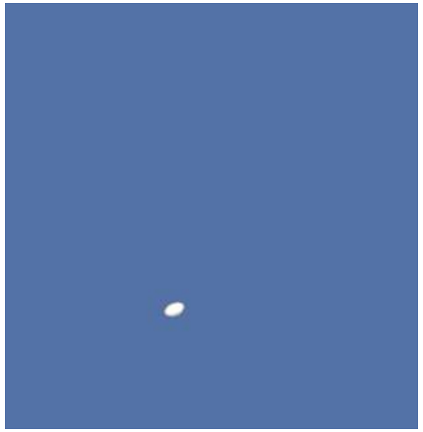
$E = 1 \quad \frac{\rho_l}{\rho_s} = 0.125$

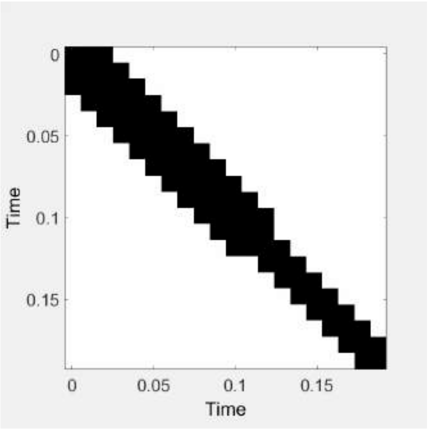




Chaotic Orbit

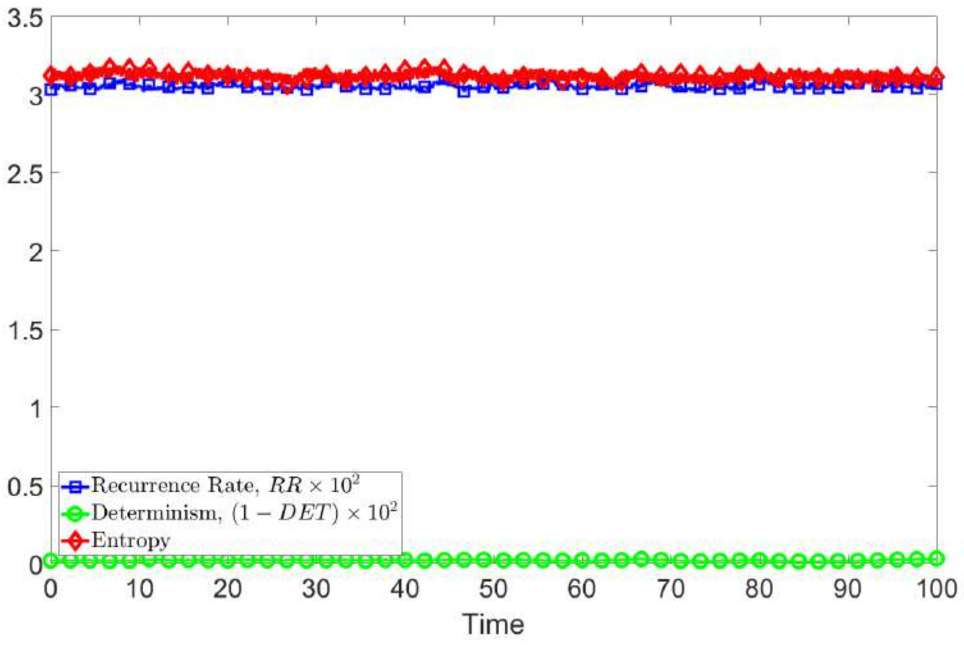
$E = 20 \quad \frac{\rho_l}{\rho_s} = 8$



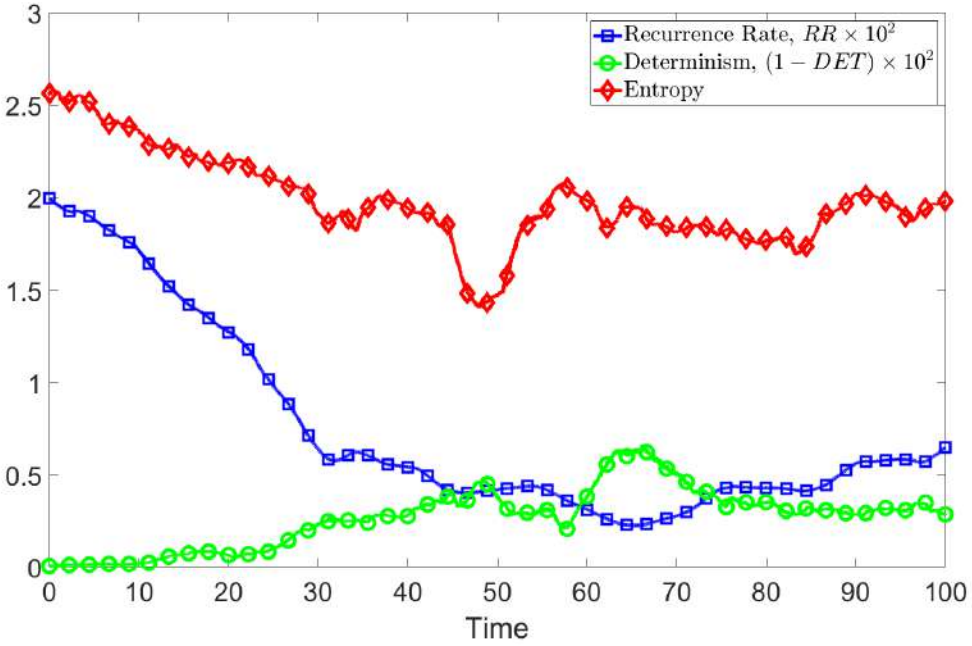


TIME DEPENDENT RQA ANALYSIS

Periodic

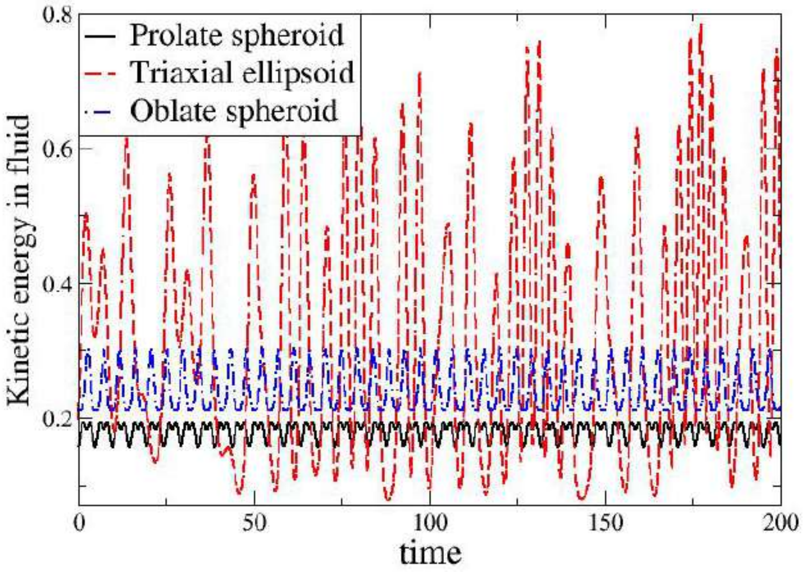


Chaos

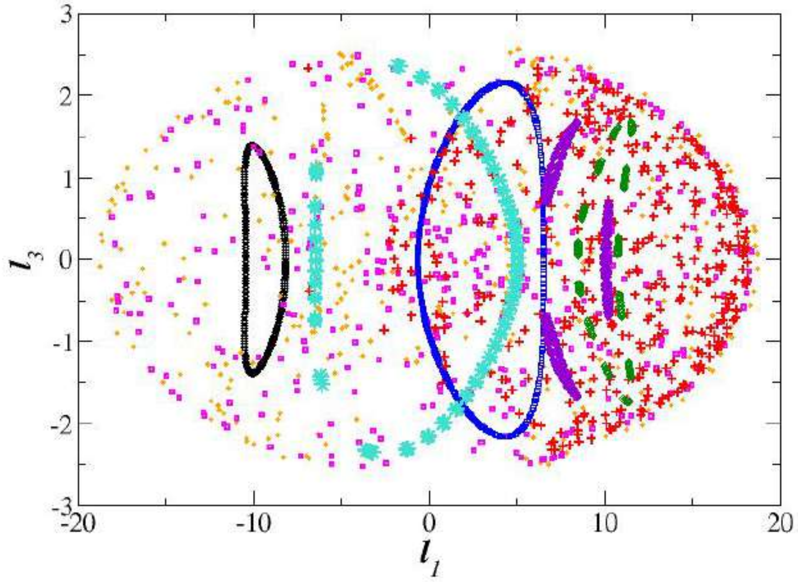


UNDERSTANDING CHAOS

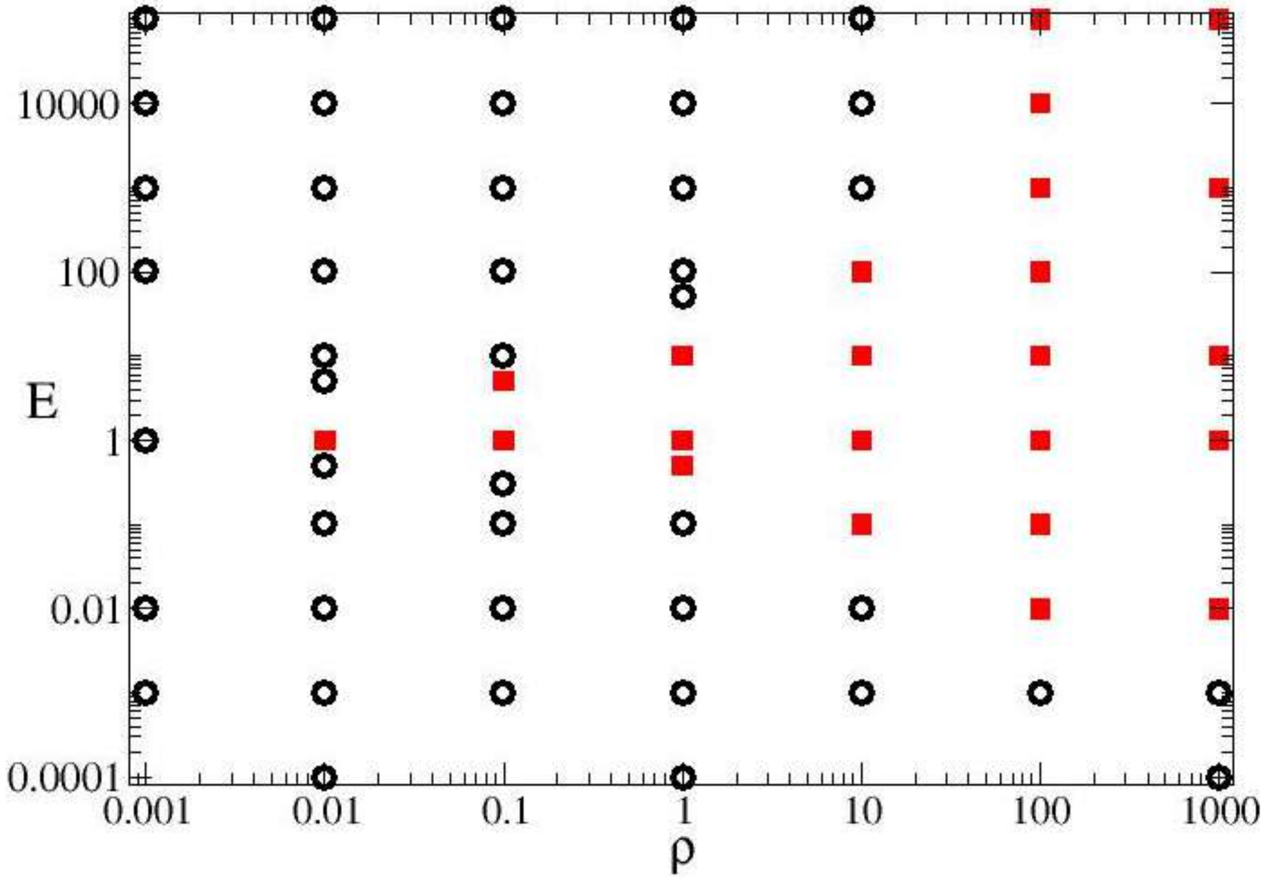
Energy Transfer

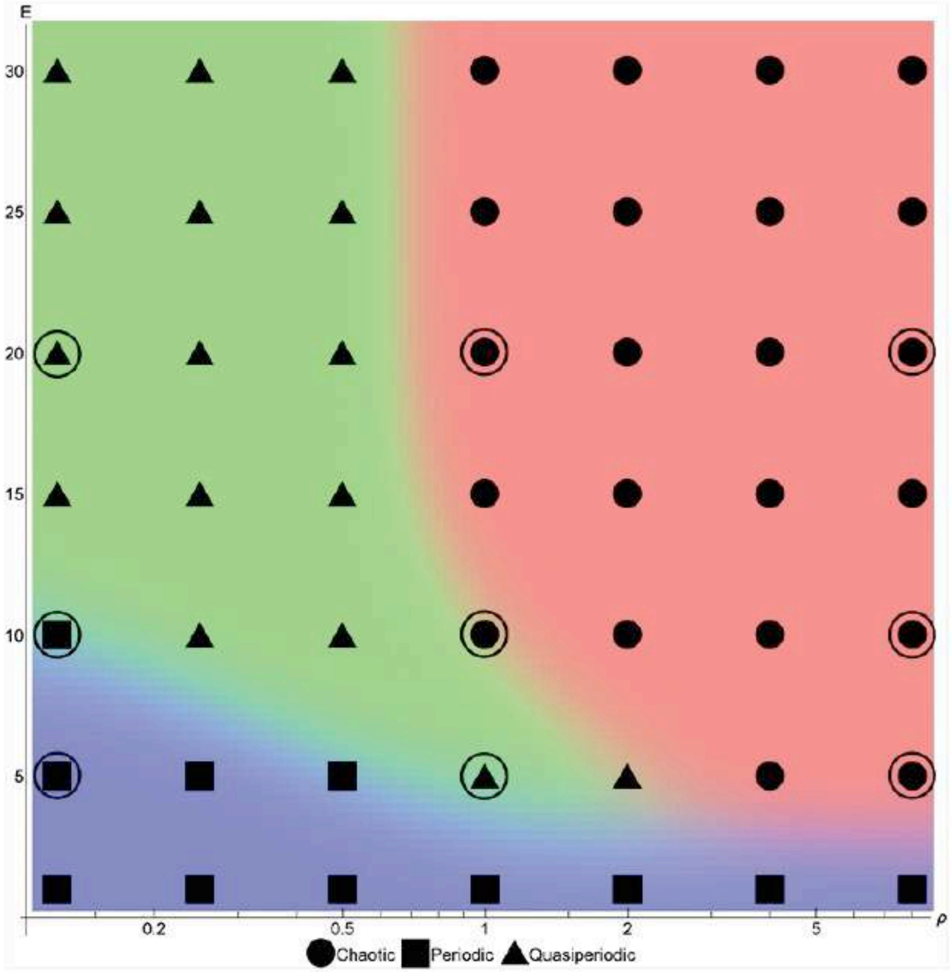


Poincare Plots

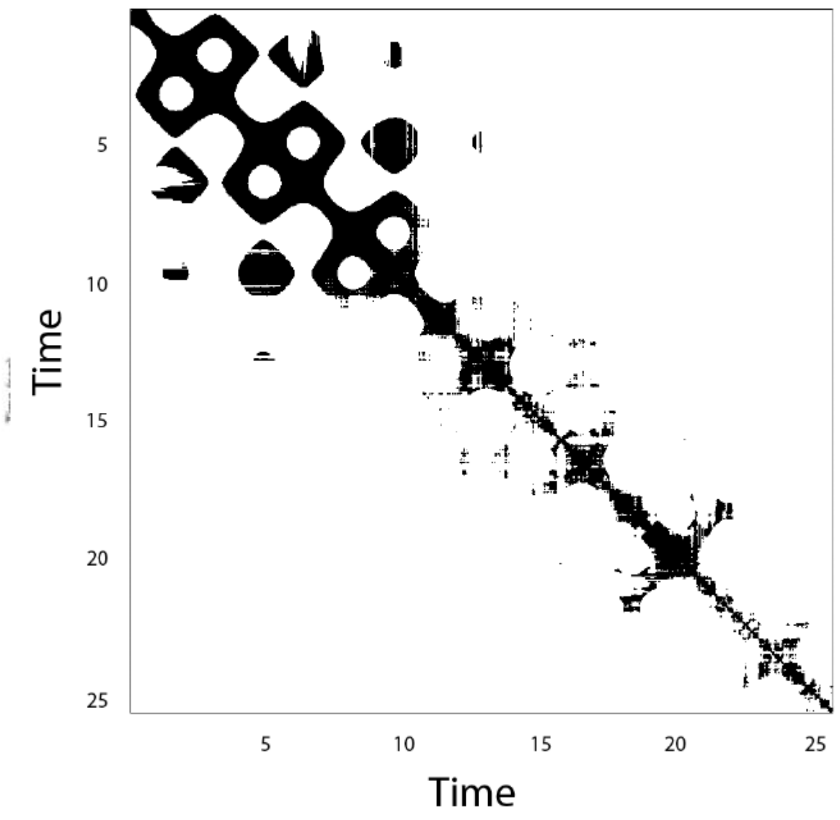


PARAMETRIC MAPS

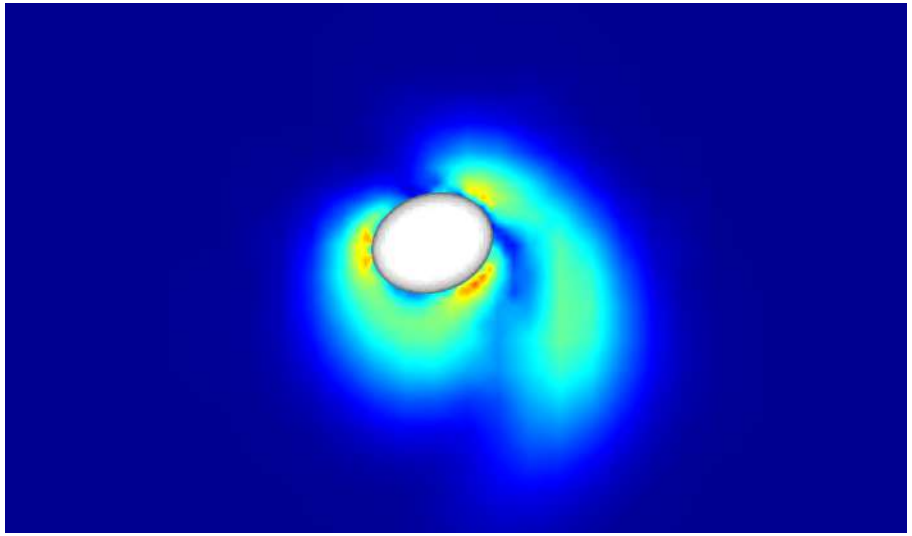




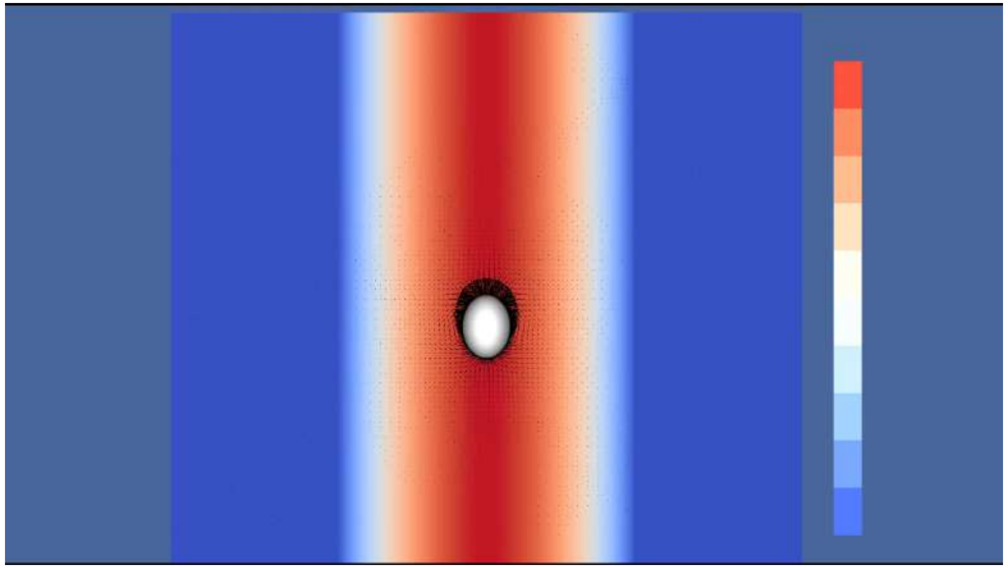
EFFECTS OF VISCOSITY ON ORBITS OF SYMMETRIC ELLIPSOIDS



Chaotic motion ***only*** under conditions of non-integrability of Kirchhoff's equations (Kozlov & Oniscenko, *Sov. Math. Dokl* 1982). Vortex shedding breaks the symmetry of the system, and allows chaotic orbits



VISCOSITY GRADIENTS AND MIXING



Passive tracers dynamics are identical to other viscous cases

Bodies move toward viscosity minima, this is consistent with work done by (Li, McKinley and Ardekani, 2015)

Viscosity gradient effects compete with geometry induced chaos

MOTION OF MULTIPLE ELLIPSOID IN INVISCID FLOW

$$\nabla^2\phi = 0$$

We know that the ϕ will have the form

$$\phi = \dot{q}_1\phi_1 + \dot{q}_2\phi_2 + \cdots + \dot{q}_n\phi_n$$

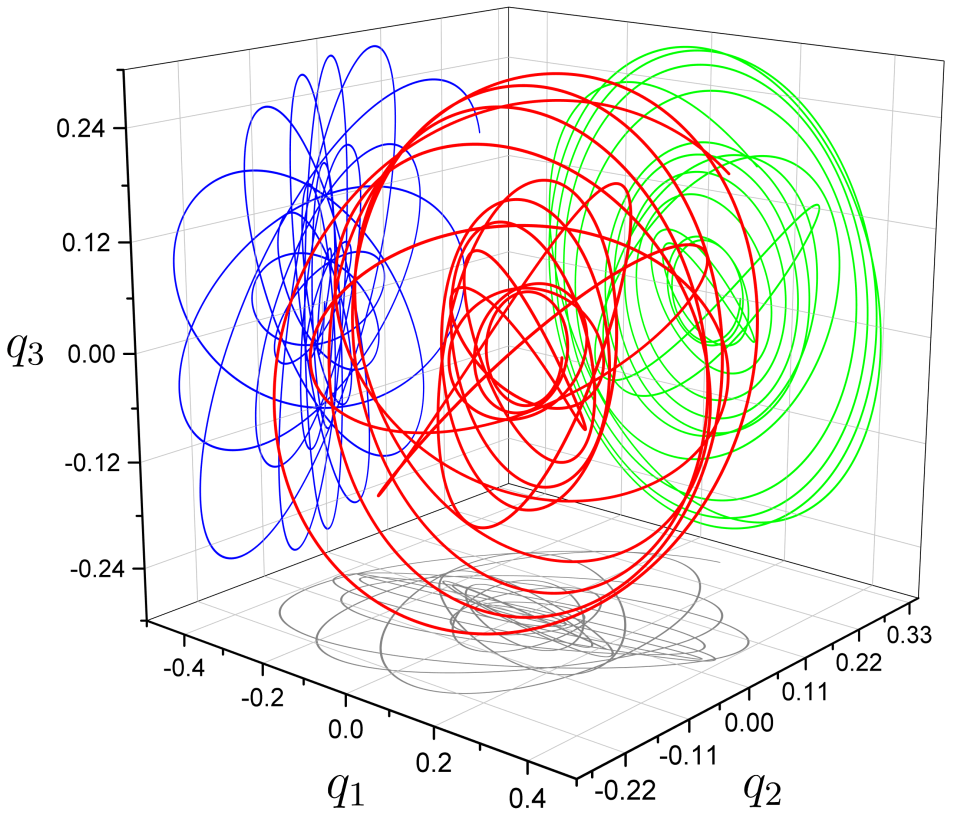
The total energy of the fluid:

$$2\boldsymbol{T} = -\rho \iint \phi \frac{\partial\phi}{\partial\,\hat{n}} dS = \sum_{i,j=1}^{n} 2\boldsymbol{A}_{i,j}\dot{q}_i\dot{q}_j$$

$$\boldsymbol{A}_{i,j} = -\rho \iint \phi_i \frac{\partial\phi_j}{\partial\hat{\boldsymbol{n}}}$$

CORRELATION BETWEEN BODIES

- Correlations in the chaotic motion of two bodies
- Diagonal lines represent times of correlations and perpendicular lines present times of anticorrelation between bodies
- Further Investigation underway



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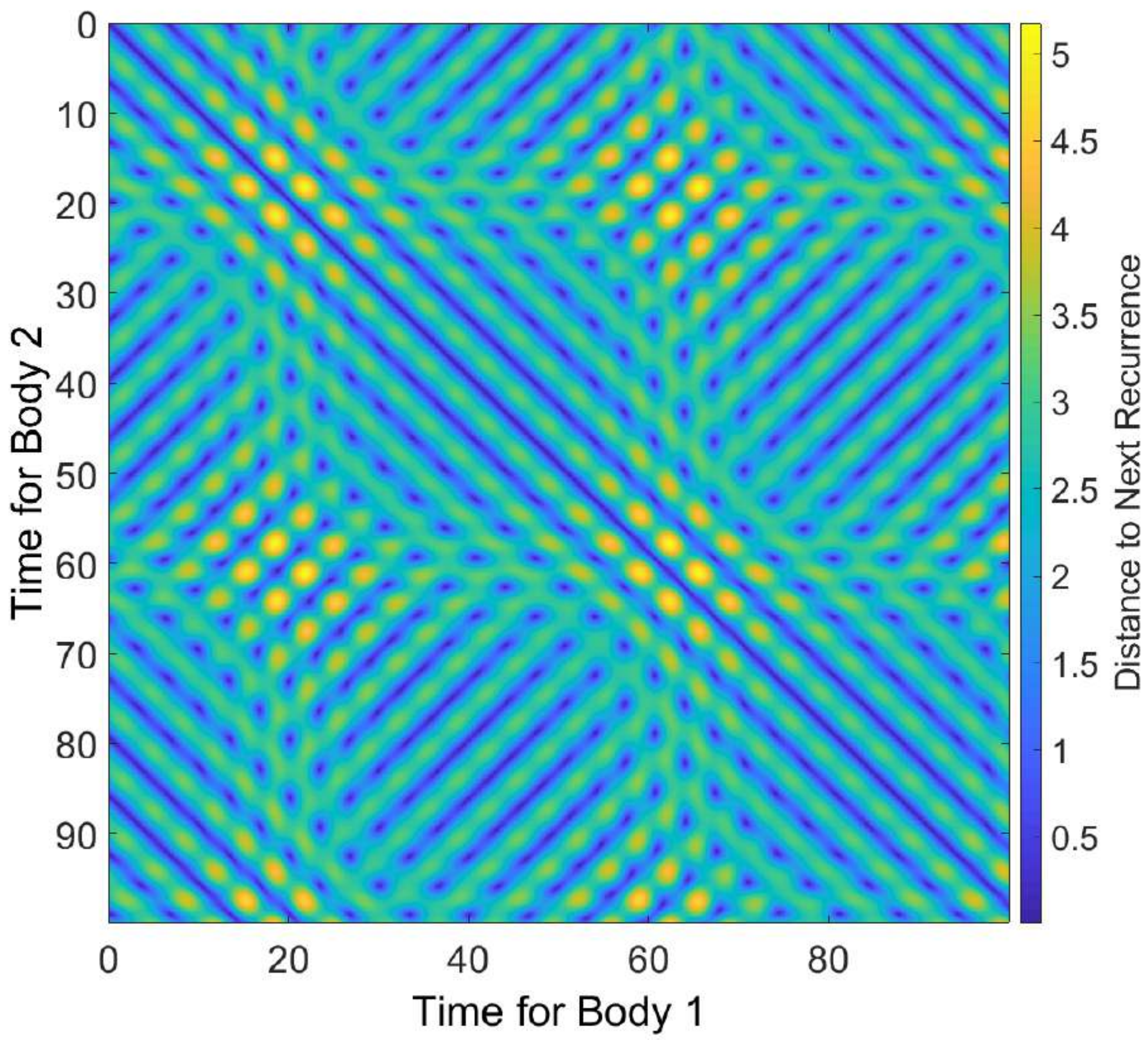

CONCLUSION & FUTURE WORK

Conclusion

Our GISS solver is capable to capture very complex motion patterns of solid-fluid.

Parametric mapping of the systems supports trend found by (Aref & Jones, 1993)

Vortex Shedding allows for the development of chaotic orbits

Future Work

Investigate the enhancement to mixing caused by the chaotic tumbling ellipsoids

Extent Kirchhoff Equation to include multiple ellipsoids

THANK YOU

For your attention